Class . Revise

3 Baar

HW

DPP

Class 11th

Kullu Bhaiya

Complex Numbers

Lecture 1

BE CONSISTENT

ALWAYS BELIEVE IN YOURSELF

मैं Is Saal 11th मैं Fodunga

CHAPTER OVERVIEW

- INTRODUCTION ✓
- CONCEPT OF IOTA ✓
- COMPLEX NUMBER ✓
- EQUALITY OF COMPLEX NUMBER ✓
- OPERATION ON COMPLEX NUMBER ✓
- CONJUGATE AND MODULES ✓
- ARGAND PLANE AND POLAR REPRESENTATION * NCERT
- MISCELLANEOUS PROBLEMS

Quaduatic Equations

$a : 1$

$b : 0$

$c : 4$

$D : b^2 - 4ac$

$= 0 - 4 \times 1 \times 4$

$: \underbrace{-16 < 0}$

↓

Roots Nahi Nikal Sakte.

$\boxed{ax^2 + bx + c : 0}$

Roots

$$x : \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\boxed{D : b^2 - 4ac}$

$D > 0 \rightarrow$ Real Distinct Roots

$D : 0 \rightarrow$ Real & Equal

$D < 0 \rightarrow \underbrace{\text{Non Real Roots}}$

→ Imaginary

Imaginary No

$x^2 : -4$

$\boxed{x : \sqrt{-4}} \rightarrow \sqrt{-ve}$

$x^2 + 4 : 0$

$\sqrt{-4}$

$\sqrt{4} \times \sqrt{-1}$

↓

$2i$

$x : ?$

Ⓐ 2

Ⓑ -2

Ⓒ Both

Angle → θ

$$\leftarrow \boxed{\sqrt{-1} = i} \rightarrow \text{iota}$$

$$\boxed{\sqrt{-a}\times\sqrt{-b} \neq \sqrt{ab}}$$

$$\boxed{\sqrt{-a}\times\sqrt{b} = \sqrt{-ab}}$$

$$\sqrt{-2} = \sqrt{2}\times\sqrt{-1} = \sqrt{2}\,i = i\sqrt{2}$$

$$\sqrt{-3} = \sqrt{3}\times\sqrt{-1} = \sqrt{3}\,i = i\sqrt{3}$$

* $\sqrt{2}\times\sqrt{-3} = \sqrt{2\times-3} = \sqrt{-6} = \boxed{\sqrt{6}\,i}$

$\sqrt{2}\times\sqrt{3}\,i = \boxed{\sqrt{6}\,i}$

* $\sqrt{-2}\times\sqrt{-3} = \sqrt{-2\times-3} = \boxed{\sqrt{6}}$

↓

$2i\times3i = \sqrt{6}\,i^2 = -1\times\sqrt{6} = \boxed{-\sqrt{6}}$

INTEGRAL POWER OF IOTA

$i^1 = i = \boxed{\sqrt{-1}}$

$(\sqrt{-1})^2 = \boxed{-1}$

$i^2 = i\cdot i = \boxed{-1}$

$i^3 = i^2\cdot i = -1\times i = \boxed{-i}$

$i^4 = i^2\cdot i^2 = -1\times-1 = \boxed{1}$

$$(\sqrt{-1})^2 = -1$$

$$i^3 = \boxed{i^2} \cdot i^1 = -1 \times i = -i$$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

$$i^3 = -i = -\sqrt{-1}$$

$$\boxed{\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}}$$

$$i^{\boxed{5}} = i^{4\times1+1}$$

$$= (i^4)^1 \cdot i^1$$

$$= (1)^1 \cdot i^1$$

$$= i$$

$$\boxed{i^n = i^{4k+r}}$$

$$\begin{array}{r|l} & 1 \\ 4 & 5 \\ & \underline{4} \\ & 1 \end{array}$$

$$\begin{array}{r|l} & 9 \\ 4 & 38 \\ & \underline{36} \\ & \underline{2} \end{array}$$

$$i^{38}$$

$$i^{4\times9+\boxed{2}}$$

$$(i^4)^9 \cdot i^2 = (1)^9 \cdot i^2$$

$$= 1 \cdot (-1) = \boxed{-1}$$

$$a^m \times a^n$$

$$a^{m+n}$$

$$i^{4+1} = i^4 \cdot i^1$$

Quadratic Equa

$i^{4\times5+3} = i^{4\times5}\cdot i^3$

$4\overline{)19}$ = 4, 16, $\boxed{3}$

$2^{-3} = \frac{1}{2^3}$

Rationalisation

Evaluate the following:

(i) $\boxed{i^{135}}$

$i^{4\times33+3}$

$(i^4)^{33}\cdot i^3$

$(1)^{33}\cdot(-i)$

$1\times(-i)$

$= -i$

(ii) i^{19}

$= i^3$

$= -i$

$i^{4\times249+3}$

$(i^4)^{249}\cdot i^3$

$(1)^{249}\cdot(-i)$

(iii) i^{-999}

$\frac{1}{i^{999}} = \frac{1}{i^3}$

$\Rightarrow \frac{1}{\boxed{-i}}\times\frac{i}{i}$

$\Rightarrow \frac{i}{-i^2}$

$\Rightarrow \frac{i}{-(-1)}$

$\Rightarrow \frac{i}{1} = \boxed{i}$

$\boxed{4}\overline{)999}$ = 249: 8, 19, 16, 39, 36, $\boxed{3}$

(iv) $(-\sqrt{-1})^{4n+3}$, $n \in N$

$(-\sqrt{-1})^{4n+3}$

$(-i)^{4n+3}$

$(-i)^{4n}\cdot i^3$

$[(-i)^{\boxed{4}}]^n\cdot i^3$

$(i^4)^n\cdot i^3$

$(1)^n\cdot i^3$

$1\times i^3 = \boxed{-i}$

$(-2)^{\boxed{4}} = 2^4$

$= 16$

$(-x)^4 = x^4$

$4\overline{)19}\,4$, 16, $③$

$\frac{1}{i^{25}}$

$4\overline{)25}\,6$, 24, 1

$\frac{2}{-5} = \frac{-2}{5}$

Show that:

(i) $\left\{i^{19}+\left(\frac{1}{i}\right)^{25}\right\}^{2}=-4$

$\Rightarrow \left[i^{3}+\frac{1}{i}\right]^{2} \Rightarrow \left[-i+\frac{1}{i}\times\frac{i}{i}\right]^{2} = \left[-i+\frac{i}{i^{2}}\right]^{2}$

$\Rightarrow \left[-i+\frac{i}{-1}\right]^{2}$

$\Rightarrow \left[-i-i\right]^{2}$

$\Rightarrow \left[-2i\right]^{2}$

$\Rightarrow (-2)^{2}\times(i)^{2}$

$\Rightarrow 4\times i^{2}$

$\Rightarrow 4\times(-1) = -4$

$4\overline{)17}\,4$, 16, 1

(ii) $\left\{i^{17}+\left(\frac{1}{i}\right)^{34}\right\}^{2}=\boxed{-2i}$

(iii) $\left\{i^{18}+\left(\frac{1}{i}\right)^{24}\right\}^{3}=0$ ✓

(iv) $i^{n}+i^{n+1}+i^{n+2}+i^{n+3}=0$ *for all n* N → JEE Mains

$$2^{x+y} = 2^x \cdot 2^y$$

$$i^n + i^{n+1} + i^{n+2} + i^{n+3}$$

$$i^n + i^n \cdot i^1 + i^n \cdot i^2 + i^n \cdot i^3$$

$$i^n\left[1 + i + i^2 + i^3\right]$$

$$i^n\left[\cancel{1} + \cancel{i} \cancel{-1} \cancel{-i}\right]$$

$$i^n \times 0$$

$$\underline{0}$$

$$\begin{array}{r} 4 \\ 4\overline{)18} \\ 16 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 6 \\ 4\overline{)24} \\ 24 \\ \hline 0 \end{array}$$

$$\left\{ i^{18} + \left(\frac{1}{i} \right)^{24} \right\}^3$$

$$\left\{ i^{2} + \frac{1}{i^{24}} \right\}^3$$

$$\left\{ -1 + \frac{1}{i^{0}} \right\}^3$$

$$\left\{ -1 + \frac{1}{1} \right\}^3$$

$$(\cancel{-1}\cancel{+1})^3 = 0$$

$$x^0 = 1$$

$$2^0 = 1$$

$$i^0 = 1$$

$$\begin{array}{r|l} & 4 \\ \hline 4 & 17 \\ & 16 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} & 8 \\ \hline 4 & 34 \\ & 32 \\ \hline & 2 \end{array}$$

$$\left[i^{17} + \left(\frac{1}{i}\right)^{34}\right]^2$$

$$\left[i^{1} + \frac{1}{i^{34}}\right]^2$$

$$\left[i^{1} + \frac{1}{i^{2}}\right]^2$$

$$\left[i + \frac{1}{-1}\right]^2$$

$$(i-1)^2$$

$$i^2 + 1^2 - 2 \times i \times 1$$

$$\cancel{-1} + \cancel{1} - 2i$$

$$\boxed{-2i}$$

Evaluate $\sum_{n=1}^{13}(i^n + i^{n+1})$, where n ∈ N.

$$\sum_{n=1}^{13}(i^n + i^n \cdot i)$$

$$\sum_{n=1}^{13} i^n (1+i)$$

$$(1+i)\boxed{\sum_{n=1}^{13} i^n}$$

$$(1+i)\left[i^1 + i^2 + i^3 + i^4 \cdots\cdots + i^{13}\right]$$

$$(1+i)\left[\underbrace{\cancel{i} - \cancel{1} - \cancel{i} + \cancel{1}}+ \underline{\quad\quad} \cancel{i^8} \underline{\quad\quad} \cancel{i^{12}} + i^{13}\right]$$

$$(1+i)\,i = i + i^2 = \underline{i - 1}$$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^9 = i$

$i^{10} = -1$

$i^{11} = -i$

$i^{12} = 1$

$i^{13} = i^1$

For a positive integer n, find the value of $(1-i)^n\left(1-\frac{1}{i}\right)^n$.

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$$

$$\frac{i^{584+8}+i^{584+6}+i^{584+2}+i^{584}+i^{584+4}}{i^{574+8}+i^{574+6}+i^{574+2}+i^{574}+i^{574+4}}$$

$$\frac{i^{584}\cdot i^{8}+i^{584}\cdot i^{6}+i^{584}\cdot i^{2}+i^{584}+i^{584}\cdot i^{4}}{i^{574}\cdot i^{8}+i^{574}\cdot i^{6}+i^{574}\cdot i^{2}+i^{574}+i^{574}\cdot i^{4}}$$

$$\frac{i^{584}\cancel{(i^{8}+i^{6}+i^{2}+i^{4}+1)}}{i^{574}\cancel{(i^{8}+i^{6}+i^{2}+i^{4}+1)}}$$

$$\frac{i^{584}}{i^{574}} = i^{584-574}$$

$$= i^{10}$$

$$4\overline{)10}\,2$$
$$\underline{8}$$
$$2$$

$$\Rightarrow i^{2}$$

$$= \boxed{-1}$$

$1 + i^2 + i^4 + i^6 + \ldots + i^{20}$

Express the following in the form a + ib :

(i) $(-5i)\left(\frac{1}{8}i\right)$ (ii) i^{-39} (iii) $(5i)\left(-\frac{3}{5}i\right)$

(iv) $i^9 + i^{19}$ (v) $(1-i)^4$

Find the real values of x and y, if

(i) $(3x - 7) + 2iy = -5y + (5 + x)i$

(ii) $(x + iy)(2 - 3i) = 4 + i$

Find real values of x and y for which the following equalities hold:

(i) $(1 + i)y^2 + (6 + i) = (2 + i)x$

Thank you Coodiess !!