
How’s the JOSH?

LECTURE 04

प्रारंभ BATCH

CLASS XI - PHYSICS

BASIC MATHS AND CALCULUS

RECAP

- Logarithms & Exponential

- Differentiation

* $\frac{d}{dx}(c) = 0$

* $\frac{d}{dx} x^n = n(x^{n-1})$

Topics to be Covered

- Chain Rule
- Applications of Differentiation
- Integration

Q.

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

Q.

$$y = x^2$$

$$\frac{dy}{dx} = 2x^{2-1}$$

$$= 2x$$

Q. $y = \sin(x^2)$

$$\frac{dy}{dx} = ? \quad \cos(2x)$$

Galat Hai!

Chain Rule = JCB Rule

$f(x) = (\sin x)^2$

$g(x) = \sin(x^2)$

inner Function = $\sin x = t$

outer Funtion = t^2

$\Rightarrow f(x) = t^2$

chain Rule

$$\frac{dy}{dx} = f'(x) = 2t\left(\frac{dt}{dx}\right)$$

$$= 2\sin x \frac{d}{dx}(\sin x)$$

$$= 2\sin x \cdot \cos x$$

Chain Rule = JCB Rule

$f(x) = (\sin x)^2 = \boxed{}^2$ OR t^2

- Outer function : ✓
- Inner function : $\boxed{\sin x}$ OR t

$$\frac{dy}{dx} = f'(x) = 2t\left(\frac{dt}{dx}\right)$$

$$= 2\sin x \cdot \frac{d}{dx}\sin x$$

$$= 2\sin x\cos x$$

Q. $y = \sin x^2$

inner Function $= x^2 = t$

$\longrightarrow y = \sin t$

$$\frac{dy}{dx} = \cos t \cdot \frac{d t}{dx}$$

$$= \cos(x^2) \cdot \frac{d(x^2)}{dx}$$

$$= \cos x^2 \cdot (2x)$$

$$= 2x \cos x^2$$

Q. $y = (\sin x)^2$

inner Function $= \sin x = t$

$\longrightarrow y = t^2$

$$\frac{dy}{dx} = 2t \cdot \frac{d t}{dx}$$

$$= 2 \sin x \cdot \frac{d(\sin x)}{dx}$$

$$= 2 \sin x \cdot \cos x$$

Q. $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

Q. $y = 2x + 3$

$$\frac{dy}{dx} = 2$$

Q. $y = (2x+3)^3$

inner $= 2x+3 = t$

$$\Rightarrow y = t^3$$

$$\frac{dy}{dx} = 3t^2 \frac{d(t)}{dx}$$

$$= 3(2x+3)^2 \frac{d}{dx}(2x+3)$$

$$= 3(2x+3)^2 (2)$$

$$= 6(2x+3)^2$$

Q. $y = \tan x$

$$\frac{dy}{dx} = \sec^2 x$$

Q. $y = 5x + 10$

$$\frac{dy}{dx} = 5$$

Lec-3

Q. $y = \tan(\underline{5x+10})$

inner Function $= 5x+10 = t$

$\Longrightarrow y = \tan(t)$

$$\frac{dy}{dx} = \sec^2 t \;\; \frac{dt}{dx}$$

$$= \sec^2(5x+10) \;\; \underline{\underline{\frac{d}{dx}(5x+10)}}$$

$$= \sec^2(5x+10)\,(5)$$

$$= 5\sec^2(5x+10)$$

The derivative of the function $f(x) = \ln(x^2 + 5)$.

- Outer function :
- Inner function : $x^2+5 = t$

* $y = \ln x$

$$\frac{dy}{dx} = \frac{1}{x}$$

* $y = x^2+5$

$$\frac{dy}{dx} = 2x$$

$$\Rightarrow y = f(x) = \ln(t)$$

$$\frac{dy}{dx} = f'(x) = \frac{1}{t}\frac{dt}{dx}$$

$$= \left(\frac{1}{x^2+5}\right)\frac{d}{dx}(x^2+5)$$

$$= \left(\frac{1}{x^2+5}\right)(2x)$$

$$f'(x) = \frac{2x}{x^2+5}$$

The derivative of the function $f(x) = e^{3x}$.

Assume

- Outer function :
- Inner function : $3x = t$

Q. $y = e^x$

$\frac{dy}{dx} = e^x$

Q. $y = 3x$

$\frac{dy}{dx} = 3$

$f(x) = e^t$

$f'(x) = e^t \cdot \frac{d(\cancel{t})}{dx}$

$= e^{3x} \frac{d}{dx}(3x)$

$= e^{3x}(3)$

$f'(x) = 3e^{3x}$

$\Delta \Rightarrow$ Bada

$\frac{d}{dt} \Leftarrow$ small change

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = v_{inst.}$$

$$a_{int.} = \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\text{Avg. velocity} = \frac{\Delta x}{\Delta t} = \frac{\Delta \text{Displacement}}{\Delta \text{time}}$$

$$\text{Avg. acc.} = \frac{\Delta v}{\Delta t}$$

Applications of Differentiation

1. To find the rate of change of a variable with respect to another
2. To determine the slope of the curve
3. To determine Maxima - Minima

1. To find the rate of change of a variable with respect to another.

$$v_{inst.} = \frac{dx}{dt} = \frac{d}{dt}(\text{Displacement})$$

$$a_{inst.} = \frac{dv}{dt} = \frac{d}{dt}(\text{velocity})$$

The distance travelled by an object in time t is given by s = (2.5) t^2 . The instantaneous speed of the object at t = 5 s will be....

A) 12.5m s^{-1}

B) 62.5ms^{-1}

C) 5m s^{-1}

D) 25m s^{-1}

$$S = (2.5)t^2 = \frac{5}{2}t^2$$

$$\text{inst. Speed} = \frac{d(S)}{dt} = \frac{d}{dt}(\text{Distance}) = \frac{d}{dt}\left(\frac{5}{2}t^2\right) = \frac{5}{\cancel{2}}(\cancel{2}t) = 5t$$

$$v_{\text{int.}\,(t=5\text{ sec})} = 5\times5 = 25\ m/s$$

प्रारंभ BATCH

The distance travelled by an object in time t is given by $s = (2.5)\ t^2$. The instantaneous speed of the object at $t = 5$ s will be.... [JEE Mains 2023]

A) $12.5 \mathrm{m\ s^{-1}}$

B) $62.5 \mathrm{ms^{-1}}$

C) $5 \mathrm{m\ s^{-1}}$

D) $25 \mathrm{m\ s^{-1}}$

ANSWER

(D) $25\ \mathrm{ms^{-1}}$

The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$m. The magnitude of velocity of the particle at t = 2s will be ..

A) 10 ms^{-1}

B) 14 ms^{-1}

C) 16 s^{-1} m/s

D) 06 ms^{-1}

$v = \frac{dx}{dt} = 10t - 4$

$v_{(t=2)} = 10(2) - 4$

$= 16$ m/s

The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$m. The magnitude of velocity of the particle at $t = 2$s will be .. [JEE Mains 2023]

A) 10 ms^{-1}

B) 14 ms^{-1}

C) 16 s^{-1} m/s

D) 06 ms^{-1}

ANSWER

(c) 16 ms^{-1}

A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = (t^3 - 6t^2 + 20t + 15)$. The velocity of the body when its acceleration becomes zero is.....

A) 4m/s

B) 8 m/s

C) 10 m/s

D) 6 m/s

HW

A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = (t^3 - 6t^2 + 20t + 15)$. The velocity of the body when its acceleration becomes zero is..... [JEE Mains 2024]

A) 4m/s

B) 8 m/s

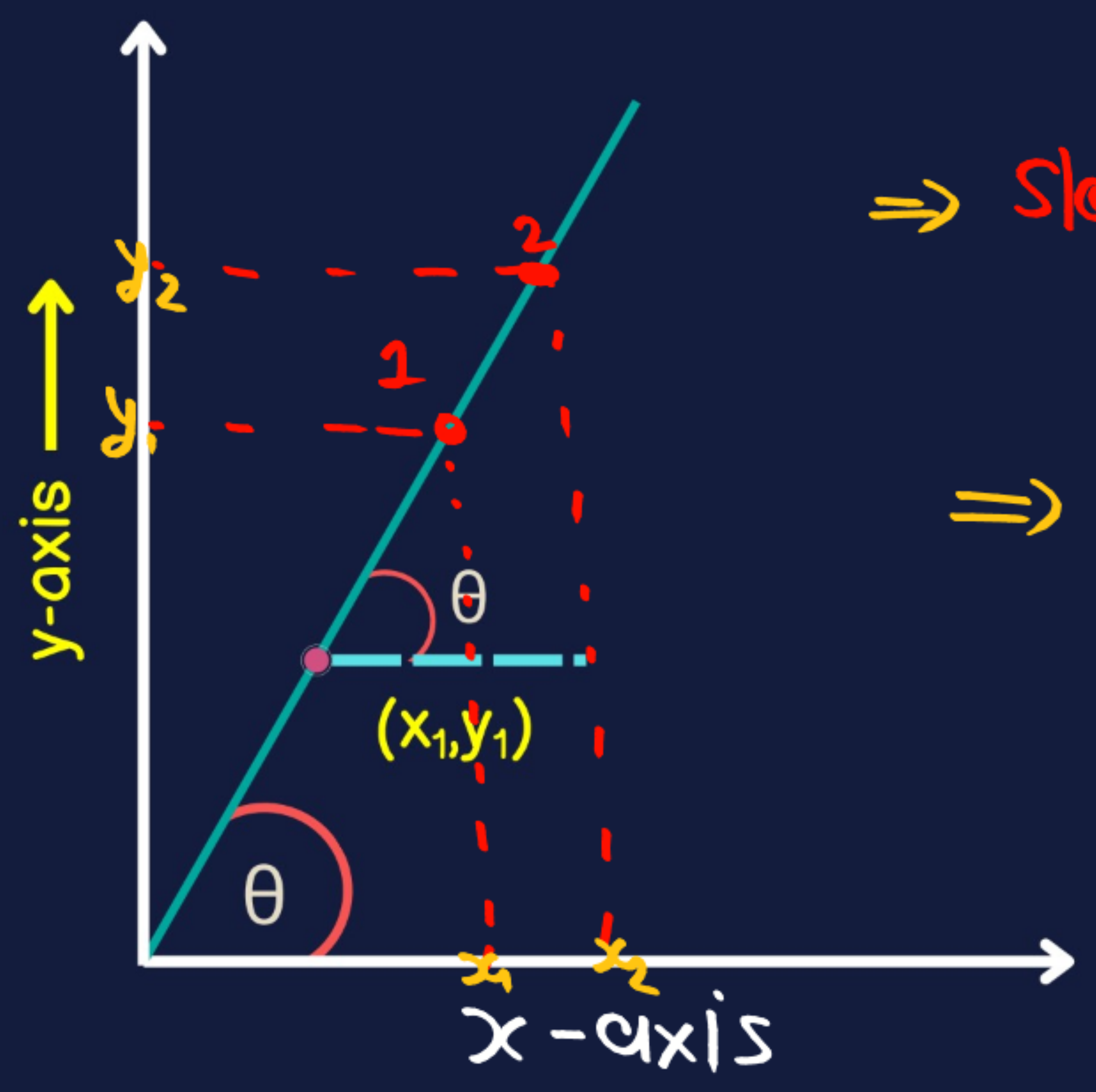
C) 10 m/s

D) 6 m/s

ANSWER

(b) 8 ms^{-1}

2. To determine the slope of the curve



$\Rightarrow$ Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x} = \tan\theta$

$\Rightarrow$ For small change $\Delta \to d$

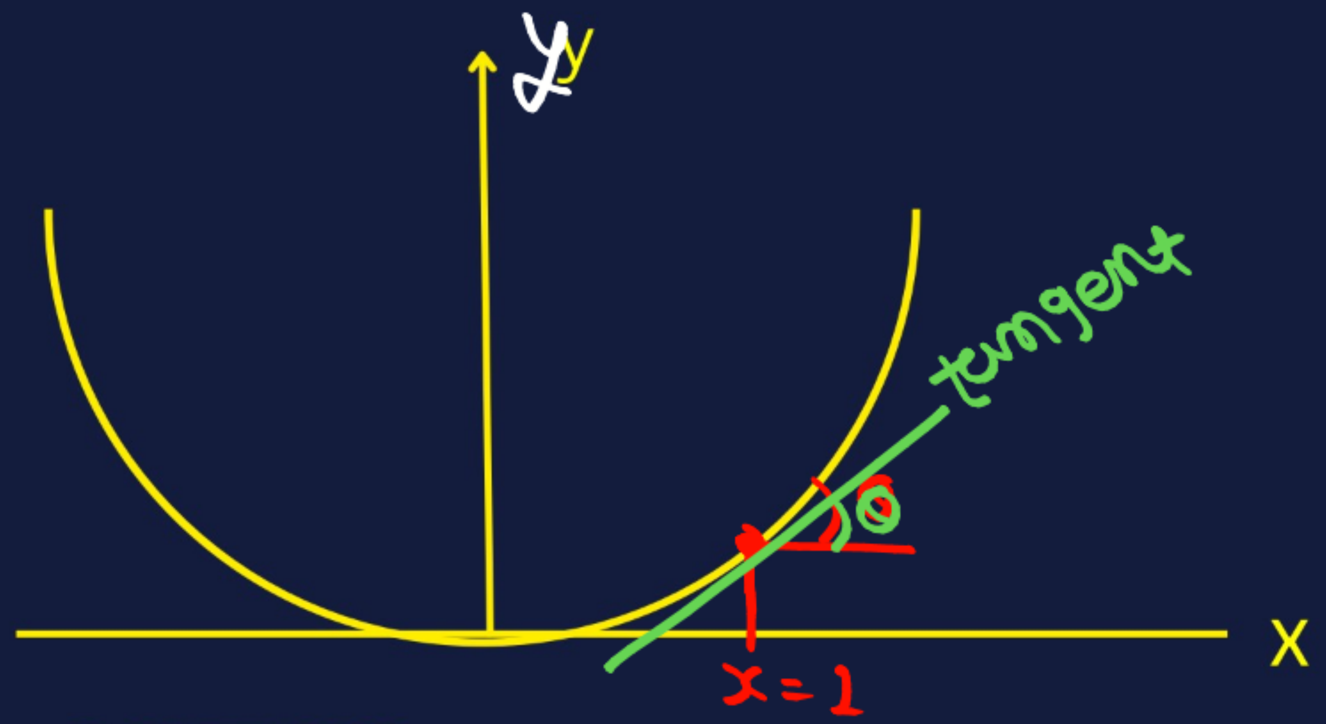
slope(m) $= \dfrac{dy}{dx} = \tan\theta$

$\theta < 90 \Rightarrow m = (+)$

$\theta > 90^\circ \Rightarrow m = (-)$

find the slope of the curve at a point $x = 1$

$y = x^2$



SOLUTION

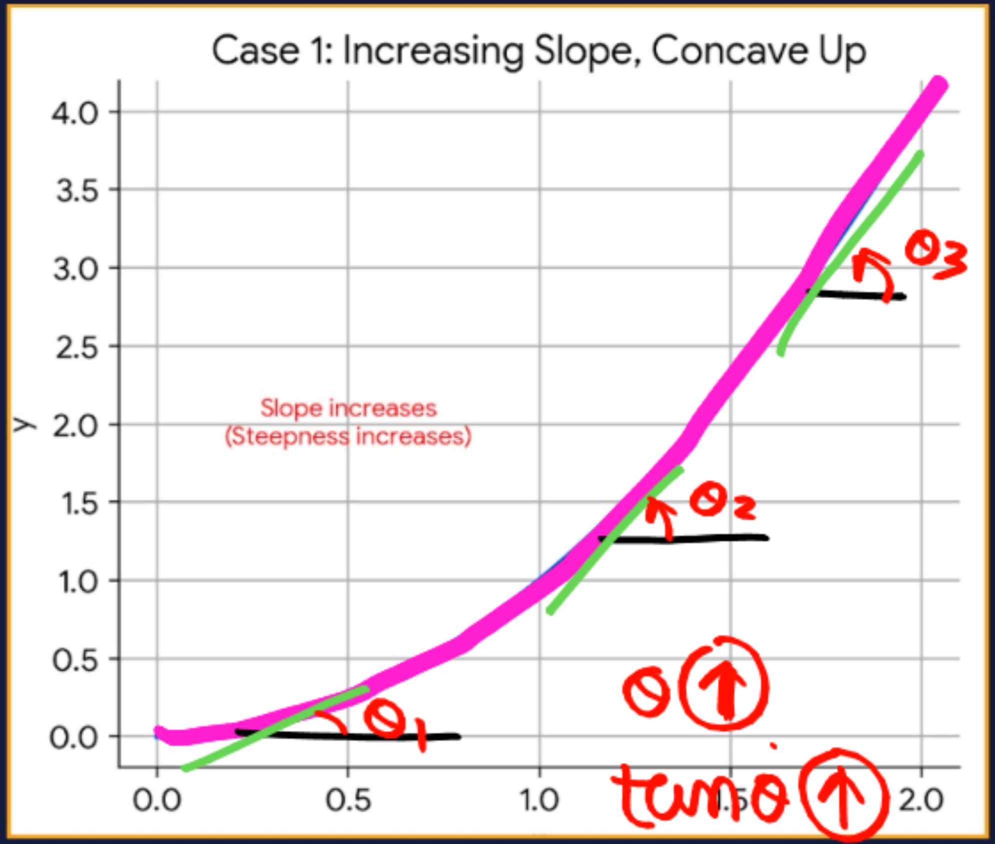
$m = \tan\theta = 1$

$$y = x^2$$

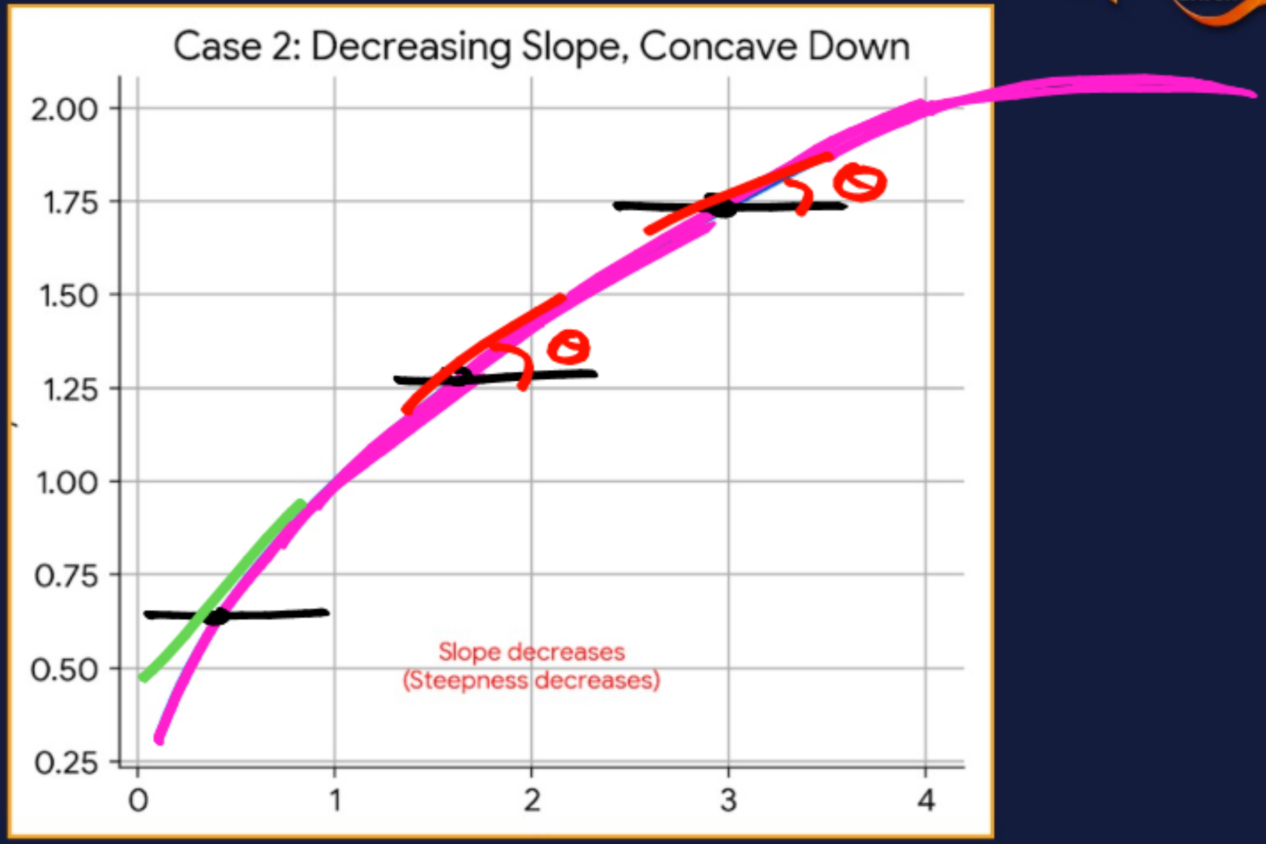
$$\frac{dy}{dx} = 2x$$

$$\text{Slope}\,(m) = \frac{dy}{dx} = 2x$$

$$m_{(x=1)} = 2(1) = 2$$



Variable Slope (PCM)



Case 1: Increasing Slope

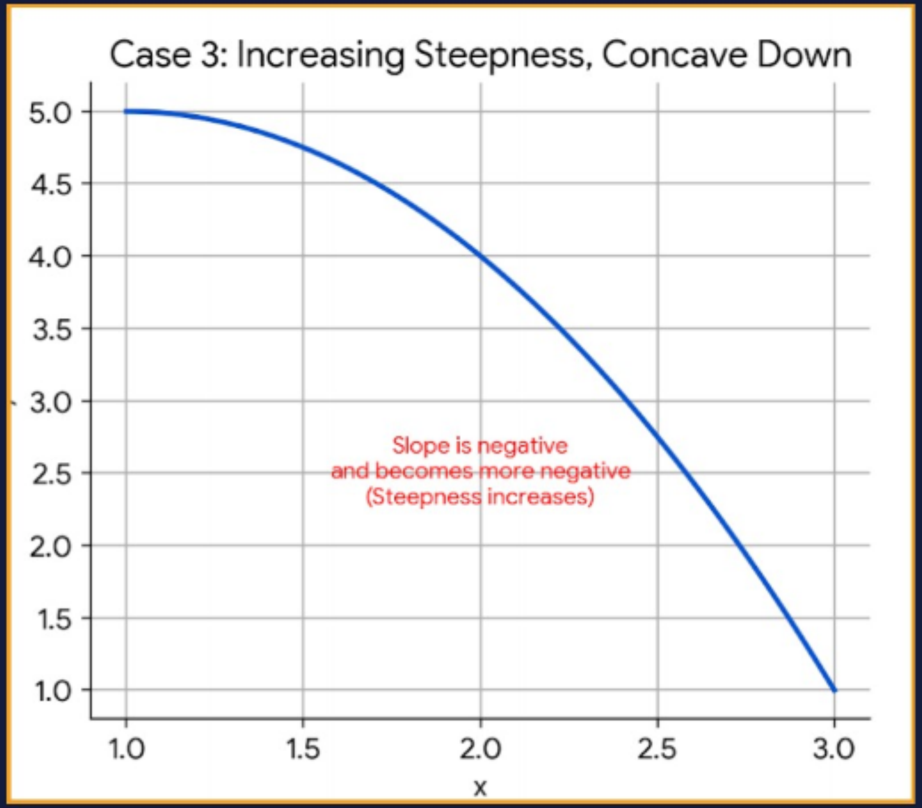
Graph: Curve bending upward (concave up).
Slope behaviour: Increasing

Interpretation: At lower levels slope is easy (gentle). As you go up, the slope becomes tougher.

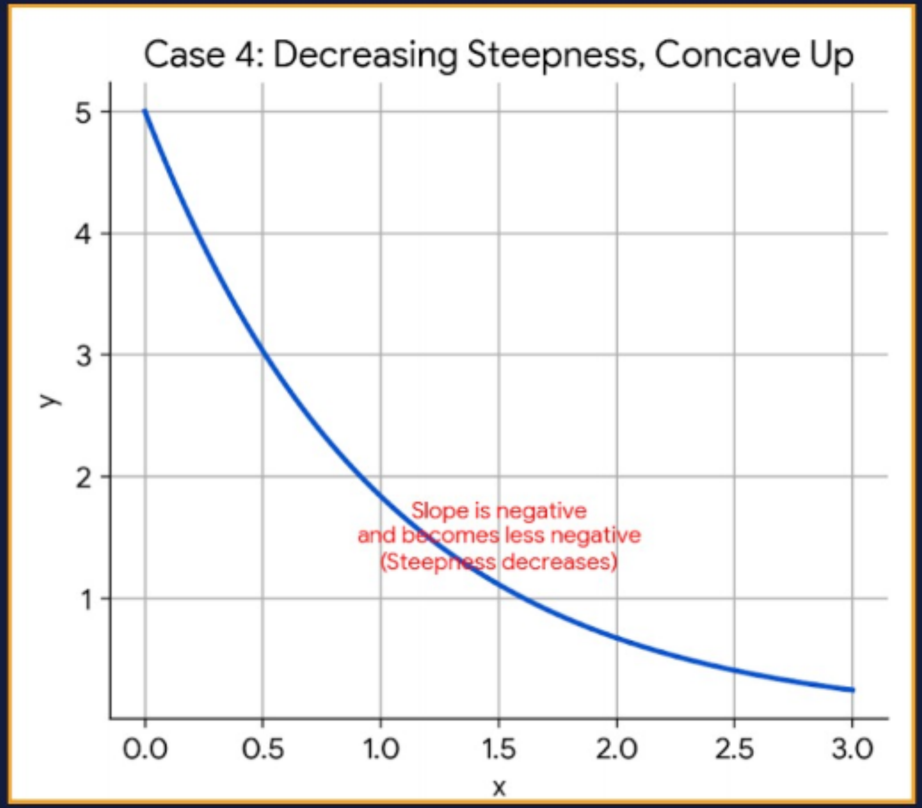
Case 2: Decreasing Slope

Graph: Curve bending downward (concave down).
Slope behaviour: Decreasing

Interpretation: Starts with a higher slope and ends with a lower slope.



(PCM)



Case 3: Increasing Slope (Inverse)

Graph: Decreasing curve starting from a high.
Slope behaviour: Increasing (in steepness as it descends).

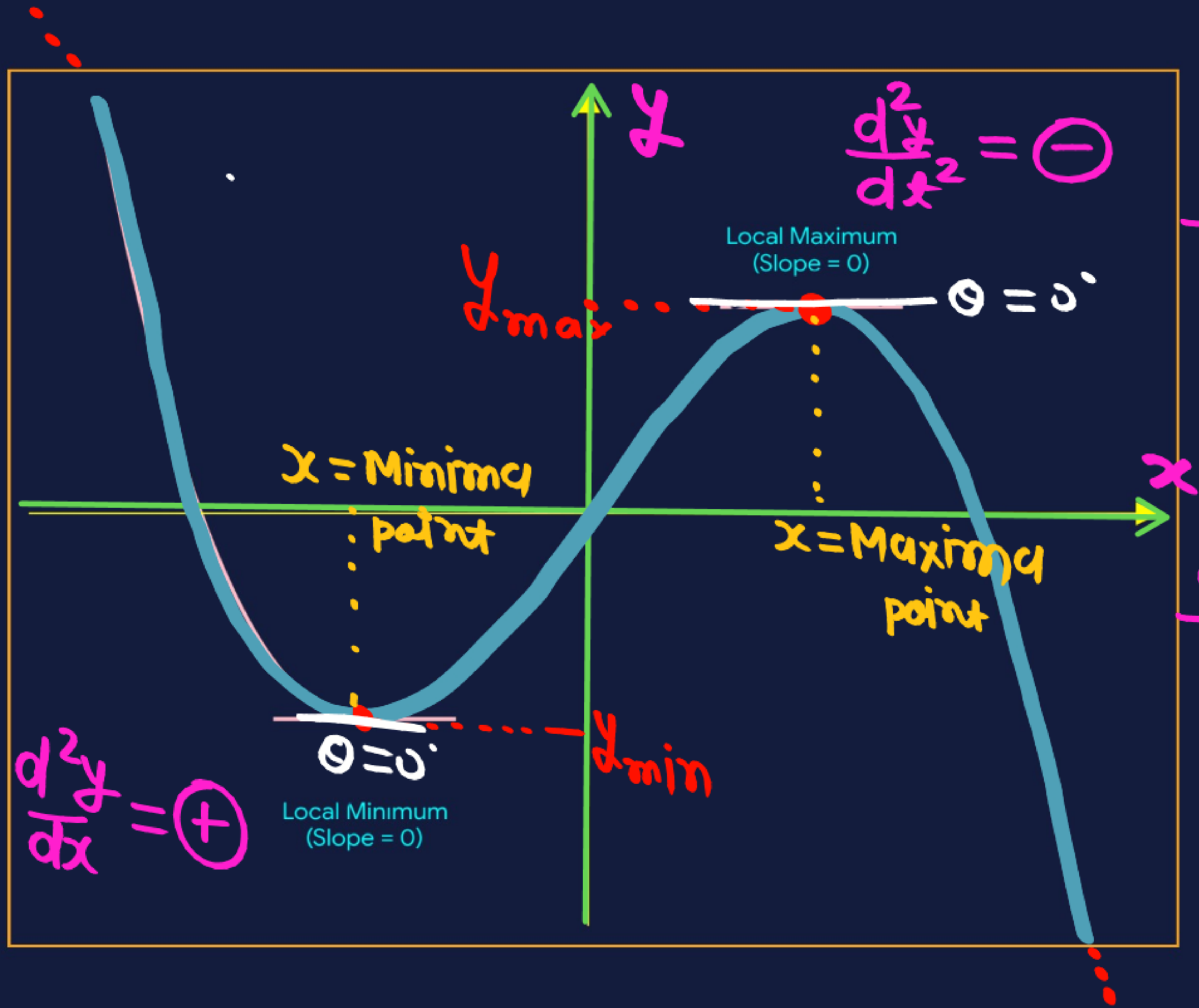
Interpretation: At higher levels slope is easy (gentle). As you go lower, the slope becomes tougher.

Case 4: Decreasing Slope (Inverse)

Graph: Decreasing curve that flattens
Slope behaviour: Steepness Decreasing

Interpretation: Starts with a higher slope and ends with a lower slope.

3. To determine Maxima - Minima



$y = f(x)$

condition - 1

Maxima OR Minima $\Rightarrow m = \tan\theta = 0$

$\Rightarrow \boxed{\frac{dy}{dx} = 0}$ $\theta = 0°$

condition - 2

Maxima $\Rightarrow \frac{d^2y}{dx^2} = \ominus$

Minima $\Rightarrow \frac{d^2y}{dx^2} = \oplus$

Q(1)

$$y = 3x^2 + 10x$$

$$\boxed{\frac{d}{dx}}$$

$$\frac{dy}{dx} = y' = 3(2x) + 10 = 6x + 10$$

(Double Diff^n.)

$$\boxed{\frac{d}{dx}} \rightarrow y'' = \frac{d^2y}{dx^2} = 6$$

Q(2)

$$y = 5x^5 + 2x^2$$

$$y' = 5(5\underline{\underline{x^4}}) + 2(2x)$$

$$y'' = 25(4x^3) + 4(1)$$

$$\therefore y'' = 100x^3 + 4$$

Ques. $y = x^2 + 2x + 2$ find min value of y ?

$y = (-1)^2 + 2(-1) + 2 = 1 - 2 + 2 = 1 = \underline{\text{Ans.}}$

SOLUTION

$$y = x^2 + 2x + 2$$

① $\frac{dy}{dx} = y' = 2x + 2$

② $\frac{d^2y}{dx^2} = y'' = 2 = +$

Step-1 Maxima/Minima

$$\frac{dy}{dx} = 0$$

$\therefore 2x + 2 = 0 \Rightarrow \boxed{x = -1}$ → Maxima ✗ / → Minima ✓

Step-2 $\left(\frac{d^2y}{dx^2}\right) = - \Rightarrow$ Maxima

$\left(\frac{d^2y}{dx^2}\right) = + \Rightarrow$ Minima

Ques. $y = -2x^2 + 8x + 3$ find min / max value of y ?

SOLUTION

$$y = -2x^2 + 8x + 3$$

$$\frac{dy}{dx} = -4x + 8$$

$$\frac{d^2y}{dx^2} = -4 \Rightarrow \ominus$$

Step-1

$$\frac{dy}{dx} = 0$$

$$-4x + 8 = 0$$

$$\boxed{x = 2}$$

Maxima ✓ Minima ✗

Step-2

$$\frac{d^2y}{dx^2} \begin{cases} \oplus \Rightarrow \text{Minima} \\ \ominus \Rightarrow \text{Maxima} \checkmark \end{cases}$$

$$y_{max} = -2x^2 + 8x + 3$$

$(x=2)$

$$= -8 + 16 + 3$$

$$= 8 + 3$$

$$y_{max} = 11$$


How’s the JOSH?

Q.

Mass = M

Let $m_1 = m$, $m_2 = (M-m)$

$F = \text{Max}$

JEE Mains.

$m_1 = ? = m = M/2$

$m_2 = ? = M - m = \frac{M}{2}$

Ans.

$m \xleftrightarrow{r} (M-m)$

$$F = \frac{G(m)(M-m)}{r^2}$$

$$F = \frac{G}{r^2}\left(M \cdot m - m^2\right)$$

Step-1

$$\frac{dF}{dm} = 0$$

$$\frac{G}{r^2}(M-2m) = 0$$

$$M - 2m = 0$$

$$\boxed{m = M/2}$$

$m_1 \xleftrightarrow{r} m_2$

$$F = \frac{G m_1 m_2}{r^2}$$

$$\frac{d}{dm}\left(Mm - m^2\right)$$

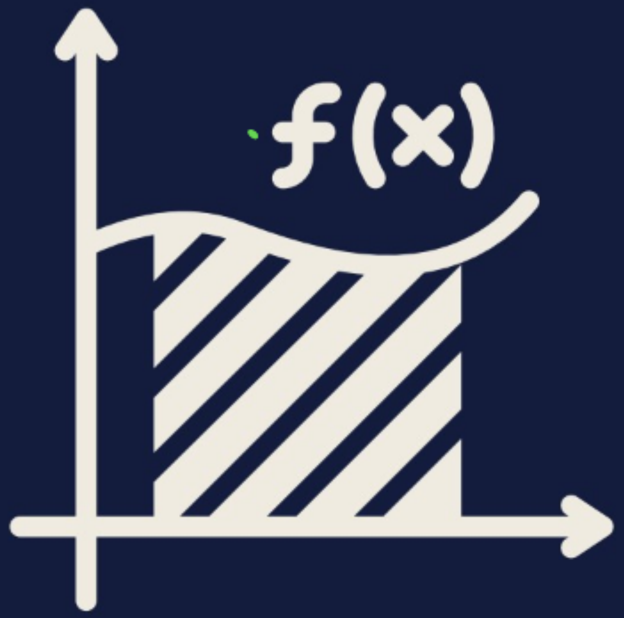
$$= M(1) - (2m)$$

$$= M - 2m$$

NEXt LECTURE GOAL

- Integration

To calculate area under curve



To Calculate the average value of function


SWAHA

STAY CONNECTED

KEEP LEARNING

Thank You