Always Believe In Yourself

$\frac{1}{\text{Cosec}}$ dor

Class 11th

Indian Army op

Complex Numbers

Lecture 6

CHAPTER OVERVIEW

- INTRODUCTION
- CONCEPT OF IOTA
- COMPLEX NUMBER
- EQUALITY OF COMPLEX NUMBER
- OPERATION ON COMPLEX NUMBER
- CONJUGATE AND MODULES.
- ARGAND PLANE AND POLAR REPRESENTATION
- MISCELLANEOUS PROBLEMS

Square Root → Many NDA

प्रारंभ BATCH

Next Toppers

$z = 2$ $\quad z\cdot\bar z = |z|^2$ $\quad |\beta|^2 = 1$

If α and β are different complex numbers with |β|=1, find $\left|\dfrac{\beta-\alpha}{1-\bar\alpha\beta}\right|$.

$$\boxed{z = \frac{\beta-\alpha}{1-\bar\alpha\beta}}$$

$$\bar z = \overline{\left(\frac{\beta-\alpha}{1-\bar\alpha\beta}\right)}$$

$$\bar z = \frac{\bar\beta-\bar\alpha}{1-\bar{\bar\alpha}\bar\beta}$$

$$\boxed{\bar z = \frac{\bar\beta-\bar\alpha}{1-\alpha\bar\beta}}$$

$$|z|^2 = z\cdot\bar z$$

$$\left|\frac{\beta-\alpha}{1-\bar\alpha\beta}\right|^2 = \left[\frac{\beta-\alpha}{1-\bar\alpha\beta}\right]\left[\frac{\bar\beta-\bar\alpha}{1-\alpha\bar\beta}\right]$$

$$\Rightarrow \frac{\beta\cdot\bar\beta-\beta\cdot\bar\alpha-\alpha\bar\beta+\alpha\cdot\bar\alpha}{1-\alpha\bar\beta-\bar\alpha\beta+\alpha\bar\alpha\,\beta\bar\beta}$$

$$\Rightarrow \frac{|\beta|^2-\beta\cdot\bar\alpha-\alpha\bar\beta+|\alpha|^2}{1-\alpha\cdot\bar\beta-\bar\alpha\beta+|\alpha|^2|\beta|^2}$$

$$\frac{\cancel{1-\beta\bar\alpha-\alpha\bar\beta+|\alpha|^2}}{\cancel{1-\bar\alpha\beta-\alpha\bar\beta+|\alpha|^2}} \quad 1$$

$$\left|\frac{\beta-\alpha}{1-\bar\alpha\beta}\right|^2 = 1$$

$$\left|\frac{\beta-\alpha}{1-\bar\alpha\beta}\right| = 1$$

$$x^2 = 4$$
$$x = \pm 2$$

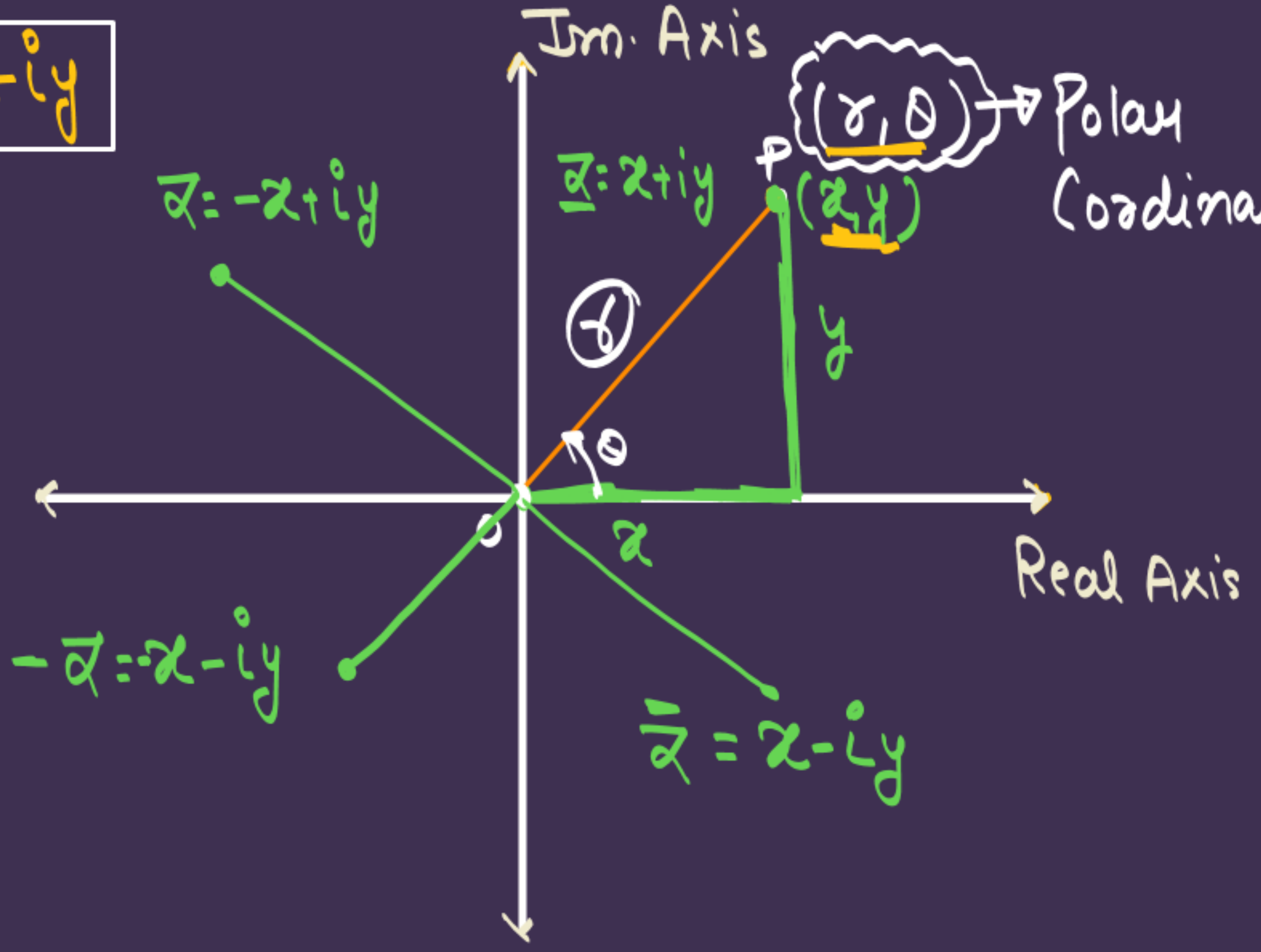
$$|x|^2 = 4$$

$$\boxed{x = 2}$$

Argand Plane (Complex Plane)

→ A plane Associated with C.N

$\boxed{z = x+iy}$



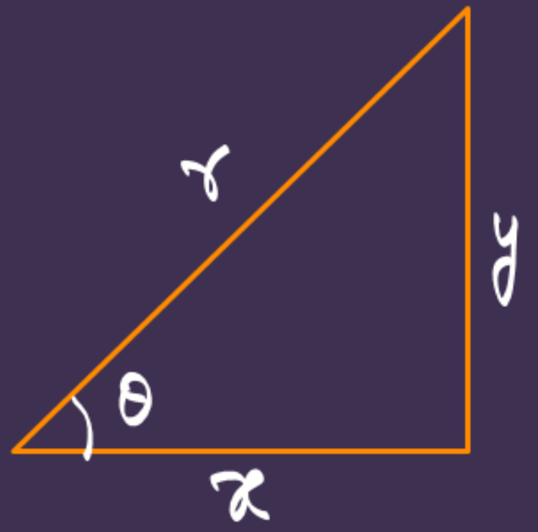
$r^2 = x^2 + y^2$

$\boxed{r = \sqrt{x^2+y^2}} = |z|$

→ Modulus of z

$\boxed{\theta \rightarrow \text{Argument/Amplitude}}$

Any R



$\sin\theta = \frac{y}{r}$ | $\cos\theta = \frac{x}{r}$

$r\sin\theta = y$ | $\boxed{r\cos\theta = x}$

$z = x+iy$

$z = r\cos\theta + i\,r\sin\theta$

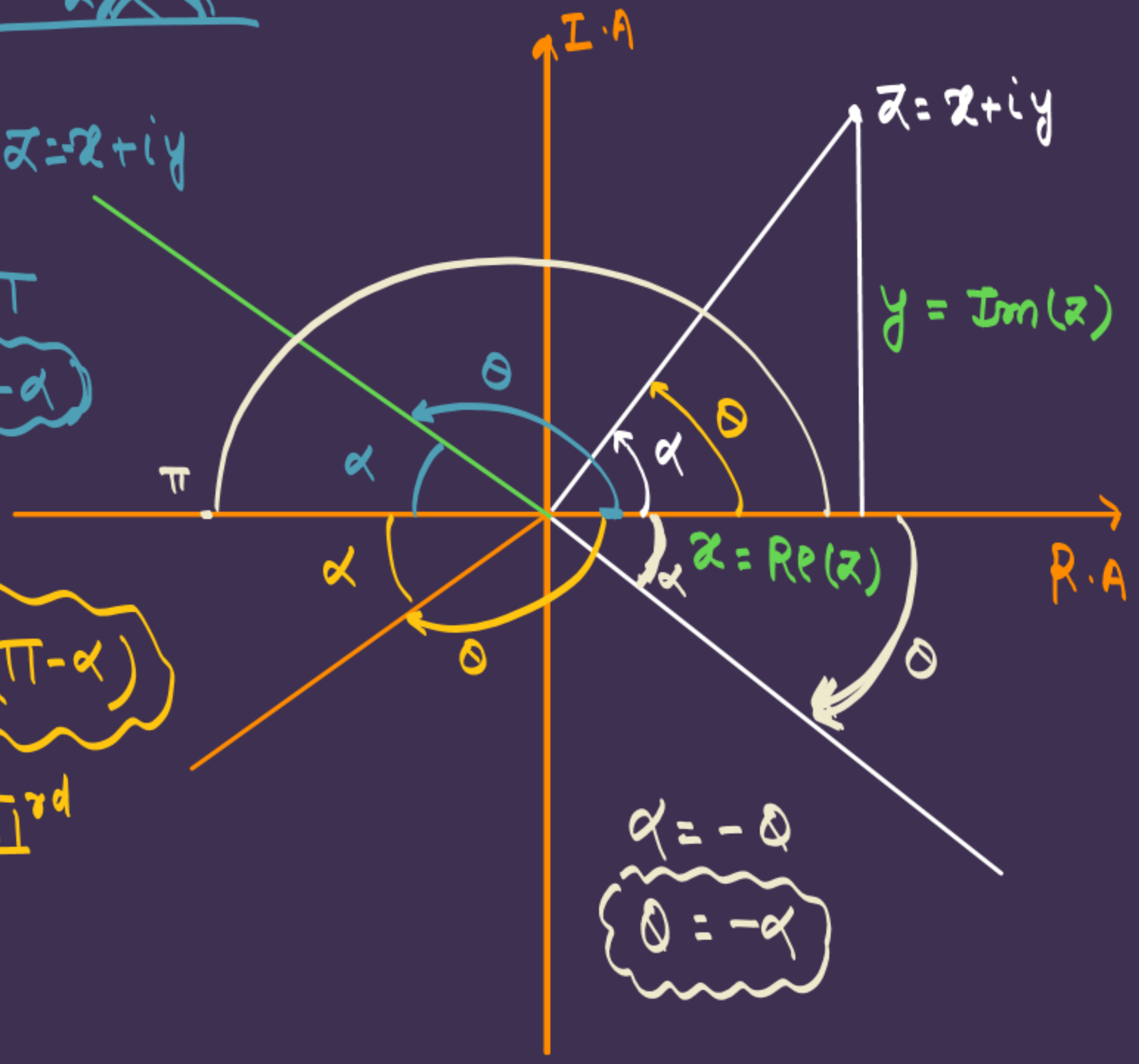
$\boxed{z = r(\cos\theta + i\sin\theta)}$ → Polar Representation

$-\pi < \theta \leq \pi$ → Principal Arg.

II^{nd} $z = x + iy$

$\alpha + \theta = \pi$

$\theta = \pi - \alpha$



$\theta = -(\pi - \alpha)$

III^{rd}

$\alpha = -\theta$

$\theta = -\alpha$

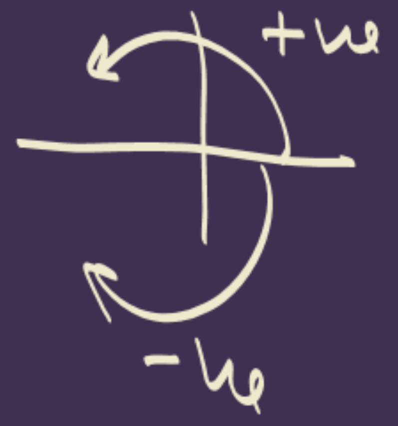
$I \rightarrow \alpha = \theta$

$-\pi < \theta \leq \pi$

$\alpha \rightarrow$ Acute Angle

$\tan\alpha = \dfrac{y}{x}$

$\tan\alpha = \left|\dfrac{Im(z)}{Re(z)}\right|$



$$\boxed{z = x + iy} \longrightarrow r(\cos\theta + i\sin\theta)$$

① $r = \sqrt{x^2 + y^2}$

② $\tan\alpha = \left|\dfrac{Im(z)}{Re(z)}\right|$

Reason

③ Quadrant Check

$I^{st} \longrightarrow \theta = \alpha$

$II^{nd} \longrightarrow \theta = \pi - \alpha$

$III^{rd} \longrightarrow \theta = -(\pi - \alpha)$

$IV^{th} \longrightarrow \theta = -\alpha$

Reason

Write the following complex numbers in the polar form:

(i) $-3\sqrt{2}+3\sqrt{2}i$

(ii) 1 + i

(iii) -1 - i

(iv) 1 - i

$45^\circ = \frac{180}{4} = \frac{\pi}{4}$

$\alpha = \tan^{-1} 1 \quad \boxed{\tan\alpha = 1}$

$z = 1 + i$

$\boxed{x = 1 \quad , y = 1}$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{1^2+1^2}$

$r = \sqrt{1+1}$

$r = \sqrt{2}$

$\tan\alpha = \left|\frac{Im(z)}{Re(z)}\right|$

$\tan\alpha = \left|\frac{1}{1}\right| = 1$

$\alpha = \frac{\pi}{4}$

Quandrant $\rightarrow$ I^{st}

$\theta = \alpha$

$\theta = \frac{\pi}{4}$

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

प्रारंभ BATCH

PKL

$\pi = 180$, $\frac{\pi}{2} = 90$, $\frac{\pi}{3} = 60$, $\frac{\pi}{4} = 45$, $\frac{\pi}{6} = 30$

Next Toppers

Write the following complex numbers in the polar form:

$* \cos(-\theta) = \cos\theta$
$* \sin(-\theta) = -\sin\theta$

(i) $-3\sqrt{2}+3\sqrt{2}i$

(ii) 1 + i

(iii) -1 - i

(iv) 1 - i

$z = -1 - i$

$x = -1$, $y = -1$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-1)^2+(-1)^2}$

$r = \sqrt{1+1}$

$r = \sqrt{2}$

$\tan\alpha = \left|\frac{Im(z)}{Re(z)}\right|$

$\tan\alpha = \left|\frac{-1}{-1}\right|$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

Quadrant $\rightarrow x = -1$, $y = -1$
$\hookrightarrow$ III rd

$\theta = -(\pi - \alpha)$

$\theta = -\left(\pi - \frac{\pi}{4}\right)$

$\theta = -\left(\frac{4\pi - \pi}{4}\right)$

$\theta = -\frac{3\pi}{4}$

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right]$

$z = \sqrt{2}\left(\cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}\right)$

Bhai → Ladke
→ Ladkiya

Write the following complex numbers in the polar form:

(i) $-3\sqrt{2} + 3\sqrt{2}i$

(ii) 1 + i

(iii) -1 - i

(iv) 1 - i

$z = -3\sqrt{2} + 3\sqrt{2}\,i$

$x = -3\sqrt{2}\ ,\ y = 3\sqrt{2}$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3\sqrt{2})^2 + (3\sqrt{2})^2}$

$= \sqrt{9\times 2 + 9\times 2}$

$= \sqrt{18+18}$

$= \sqrt{36}$

$r = 6$

$\tan\alpha = \left|\frac{Im(z)}{Re(z)}\right|$

$\tan\alpha = \left|\frac{3\sqrt{2}}{-3\sqrt{2}}\right|$

$\tan\alpha = |-1|$

$\tan\alpha = 1$

$\rightarrow \alpha = \pi/4$

Quadrant → II

$\theta = \pi - \alpha$

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$z = r(\cos\theta + i\sin\theta)$

$z = 6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

प्रारंभ BATCH

Next Toppers

$x^2 + y^2 > 0$

$(x^2-y^2)^2 = (x^2)^2 + (y^2)^2 - 2x^2y^2$

Find the square roots of the following complex numbers:

(i) 7–24i (ii) 5 + 12i

$\boxed{\sqrt{7-24i} = x+iy}$

S.O.B.S

$(x+iy)^2 = 7-24i$

$x^2+(iy)^2+2xiy = 7-24i$

$x^2-y^2+2xyi = 7-24i$

$x^2-y^2 = 7$ | $2xy = -24$

$4x^2y^2 = 576$

$x^2 - \cancel{y^2} = 7$

$x^2 + \cancel{y^2} = 25$

$2x^2 = 32$

$x = \pm 4$

$x = 4,\ y = -3$ | $x = -4,\ y = 3$

$\sqrt{7-24i} = x+iy$

$\Rightarrow 4-3i$

$\Rightarrow -4+3i$

$\pm(4-3i)$

$(x^2+y^2)^2 = (x^2)^2 + (y^2)^2 + 2x^2y^2$

$(x^2+y^2)^2 = (x^2-y^2)^2 + 2x^2y^2 + 2x^2y^2$

$\boxed{(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2}$

$(x^2+y^2)^2 = 7^2 + 576$

$(x^2+y^2)^2 = 49 + 576$

$(x^2+y^2)^2 = 625$

$\boxed{x^2+y^2 = 25}$

Thank you Coodiess !!