

ALWAYS BELIEVE IN YOURSELF

Class 11th

Complex Numbers

Lecture 4

CHAPTER OVERVIEW

MONDAY

[R & F]
[TRIGNO]

INTRODUCTION

CONCEPT OF IOTA

COMPLEX NUMBER

EQUALITY OF COMPLEX NUMBER

OPERATION ON COMPLEX NUMBER

CONJUGATE AND MODULES.

ARGAND PLANE AND POLAR REPRESENTATION

MISCELLANEOUS PROBLEMS

Square Root

Find real theta such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is purely real.

$$\sin \theta = 0$$

$$\text{Im}(z) = 0$$

$$\rightarrow \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta} \times \frac{1 + 2i \sin \theta}{1 + 2i \sin \theta}$$

$$\Rightarrow \frac{3 + 6i \sin \theta + 2i \sin \theta + 4i^2 \sin^2 \theta}{1^2 - (2i \sin \theta)^2}$$

$$\Rightarrow \frac{3 + 8i \sin \theta - 4 \sin^2 \theta}{1 - 4i^2 \sin^2 \theta}$$

$$1 - 4i^2 \sin^2 \theta$$

$$1 - (-4 \sin^2 \theta) \\ 1 + 4 \sin^2 \theta$$

$$\Rightarrow \frac{3 - 4 \sin^2 \theta + 8i \sin \theta}{1 + 4 \sin^2 \theta}$$

$$\Rightarrow \frac{3 - 4 \sin^2 \theta}{1 + 4 \sin^2 \theta} + \frac{8i \sin \theta}{1 + 4 \sin^2 \theta}$$

$$\frac{8i \sin \theta}{1 + 4 \sin^2 \theta} = 0$$

$$8i \sin \theta = 0$$

$$\sin \theta = 0$$

$$\theta = n\pi, \\ n \in \mathbb{Z}$$

If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that: $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.

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$$\overline{x+iy} = \sqrt{\frac{\overline{a+ib}}{\overline{c+id}}}$$

$$\Rightarrow x-iy = \sqrt{\frac{a-ib}{c-id}}$$

$$\Rightarrow \underline{(x+iy)} \underline{(x-iy)} = \sqrt{\frac{a+ib}{c+id}} \times \sqrt{\frac{a-ib}{c-id}}$$

$$\Rightarrow x^2 + y^2 = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$$

$$(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$$

Find the least positive value of n, if $(\frac{1+i}{1-i})^n = 1$.

$$\rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^n = 1$$

$$\Rightarrow \left[\frac{(1+i)^2}{1^2 - i^2} \right]^n = 1$$

$$\Rightarrow \left[\frac{1+i^2+2i}{1-(-1)} \right]^n = 1$$

$$\left[\frac{1-1+2i}{1+1} \right]^n = 1$$

$$\left(\frac{2i}{2} \right)^n = 1$$

$$i^n = 1$$

$$\boxed{n=4}$$

$$\begin{aligned} i^0 &= 1 \\ i^1 &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \\ i^5 &= i \\ i^6 &= -1 \\ i^7 &= -i \\ i^8 &= 1 \end{aligned}$$

If $a + ib = \frac{c+i}{c-i}$, where c is real, prove that: $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2 - 1}$.

$$\overline{a+ib} = \frac{\overline{c+i}}{\overline{c-i}}$$

$$a-ib = \frac{c-i}{c+i}$$

$$(a+ib)(a-ib) = \left[\frac{c+i}{c-i} \right] \left[\frac{c-i}{c+i} \right]$$

$$a^2 + b^2 = 1$$

$$a+ib = \frac{c+i}{c-i} \times \frac{c+i}{c+i}$$

$$a+ib = \frac{(c+i)^2}{c^2 - (i)^2}$$

$$a+ib = \frac{c^2 + i^2 + 2ci}{c^2 - (-1)}$$

$$a+ib = \frac{c^2 - 1 + 2ci}{c^2 + 1}$$

$$x \times x = x^2$$

$$a+ib = \frac{c^2 - 1}{c^2 + 1} + \frac{2ci}{c^2 + 1}$$

$$a = \frac{c^2 - 1}{c^2 + 1}, \quad b = \frac{2c}{c^2 + 1}$$

$$\frac{b}{a} = \frac{\frac{2c}{c^2 + 1}}{\frac{c^2 - 1}{c^2 + 1}} = \frac{2c}{c^2 - 1}$$

If $(x + iy)^{1/3} = a + ib$, $x, y, ab \in \mathbb{R}$. Show that $\Rightarrow (a^2 - 3b^2) - (3a^2 - b^2) = -2a^2 - 2b^2 = -2(a^2 + b^2)$

(i) $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

(ii) $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$

$$[(x + iy)^{1/3}]^3 = (a + ib)^3$$

$$\begin{aligned} (x + iy) &= a^3 + (ib)^3 + 3a \times ib(a + ib) \\ &\Rightarrow a^3 + i^3 b^3 + 3a^2 ib + 3a i^2 b^2 \\ &\Rightarrow a^3 - ib^3 + 3a^2 bi - 3ab^2 \\ &\Rightarrow \underline{a^3 - 3ab^2 + 3a^2 bi - ib^3} \end{aligned}$$

$$x = a^3 - 3ab^2$$

$$y = 3a^2 b - b^3$$

$$x = a(a^2 - 3b^2)$$

$$y = b(3a^2 - b^2)$$

$$\frac{x}{a} = a^2 - 3b^2$$

$$\frac{y}{b} = 3a^2 - b^2$$

$$\begin{aligned} \frac{x}{a} + \frac{y}{b} &= \underline{a^2 - 3b^2} + \underline{3a^2 - b^2} \\ &= 4a^2 - 4b^2 \\ &= 4(a^2 - b^2) \end{aligned}$$

If $\frac{a + ib}{c + id} = x + iy$, prove that $\frac{a - ib}{c - id} = x - iy$ and $\frac{a^2 + b^2}{c^2 + d^2} = x^2 + y^2$.

If $\overline{(a + ib)} \overline{(c + id)} \overline{(e + if)} \overline{(g + ih)} = \overline{A + iB}$ prove that
 $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

$$* \quad A + iB = (a + ib)(c + id)(e + if)(g + ih)$$

$$A - iB = (a - ib)(c - id)(e - if)(g - ih)$$

$$(A + iB)(A - iB) = \underline{(a + ib)(a - ib)} \underline{(c + id)(c - id)} (e + if)(e - if)(g + ih)(g - ih)$$

$$A^2 + B^2 = (a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2)$$

If α and β are different complex numbers with $|\beta|=1$, find $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$.

$$\alpha \cdot \bar{\alpha} = |\alpha|^2$$

$x \neq 0$

Find non-zero integral solutions of $|1 - i|^x = 2^x$.

$$x = -1$$

$$y = -1$$

$$|1 - i|^x = 2^x$$

$$\left[\sqrt{1^2 + (-1)^2} \right]^x = 2^x$$

$$\left(\sqrt{1+1} \right)^x = 2^x$$

$$(\sqrt{2})^x = 2^x$$

$$\left[(2)^{\frac{1}{2}} \right]^x = 2^x$$

$$2^{x/2} = 2^x$$

$$\frac{x}{2} = x$$

$$x = 2x$$

$$0 = 2x - x$$

$$\boxed{0 = x}$$

$$1 + i(-1) = 1 - i$$

$$|x + iy|$$

$$\Rightarrow \sqrt{x^2 + y^2}$$

Zero Non Integral Solⁿ

Thank you Goodieess !!

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PROPERTIES OF CONJUGATE

$$\rightarrow (\overline{\overline{z}}) = z$$

$$\rightarrow \boxed{z + \overline{z} = 2\operatorname{Re}(z)}$$

$$\rightarrow z - \overline{z} = 2i\operatorname{Im}(z)$$

$$\rightarrow \overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$$

$$\rightarrow \overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$$

$$\rightarrow \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

PROPERTIES OF MODULUS

$$\rightarrow |z| = |\overline{z}| = |-z|$$

$$\rightarrow |z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$\rightarrow \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$* \boxed{z \cdot \overline{z} = |z|^2}$$

$$\rightarrow |z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \cdot \overline{z_2})$$

$$\rightarrow |z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\operatorname{Re}(z_1 \cdot \overline{z_2})$$

$$|\vec{r}_1 - \vec{r}_2|^2$$

$$(\vec{r}_1 - \vec{r}_2)(\overline{\vec{r}_1 - \vec{r}_2}) = |\vec{r}_1 - \vec{r}_2|^2$$

$$(\vec{r}_1 - \vec{r}_2)(\overline{\vec{r}_1} - \overline{\vec{r}_2})$$

$$\vec{r}_1 \cdot \overline{\vec{r}_1} - \vec{r}_1 \cdot \overline{\vec{r}_2} - \vec{r}_2 \cdot \overline{\vec{r}_1} + \vec{r}_2 \cdot \overline{\vec{r}_2}$$

$$|\vec{r}_1|^2 + |\vec{r}_2|^2 - \vec{r}_1 \cdot \overline{\vec{r}_2} - \vec{r}_2 \cdot \overline{\vec{r}_1}$$

$$|\vec{r}_1|^2 + |\vec{r}_2|^2 - \vec{r}_1 \cdot \overline{\vec{r}_2} - \overline{\vec{r}_1 \cdot \vec{r}_2}$$

$$|\vec{r}_1|^2 + |\vec{r}_2|^2 - (\vec{r}_1 \cdot \overline{\vec{r}_2} + \overline{\vec{r}_1 \cdot \vec{r}_2})$$

$$|\vec{r}_1|^2 + |\vec{r}_2|^2 - 2 \operatorname{Re}(\vec{r}_1 \cdot \overline{\vec{r}_2})$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\vec{r} \cdot \overline{\vec{r}} = |\vec{r}|^2$$

$$(\vec{r}_1 - \vec{r}_2)(\overline{\vec{r}_1 - \vec{r}_2}) = |\vec{r}_1 - \vec{r}_2|^2$$

$$\overline{\vec{r}_1 - \vec{r}_2} = \overline{\vec{r}_1} - \overline{\vec{r}_2}$$

$$(\overline{\vec{r}_1 \cdot \vec{r}_2}) = \overline{\vec{r}_1} \cdot \overline{\vec{r}_2}$$

$$= \overline{\vec{r}_1} \cdot \overline{\vec{r}_2}$$

$$\vec{r} + \overline{\vec{r}} = 2 \operatorname{Re}(\vec{r})$$

$$(\vec{r}_1 + \vec{r}_2) + (\overline{\vec{r}_1 + \vec{r}_2}) = 2 \operatorname{Re}(\vec{r}_1 + \vec{r}_2)$$

$$z + \bar{z}$$

$$\Rightarrow z = \underbrace{x}_{\text{Re}(z)} + \underbrace{iy}_{\text{Im}(z)}$$

$$\bar{z} = x - iy$$

$$z + \bar{z} = \cancel{x + iy} + \cancel{x - iy}$$

$$= 2x$$

$$z + \bar{z} = 2 \operatorname{Re}(z)$$

$$z - \bar{z}$$

$$(x + iy) - (x - iy)$$

$$\cancel{x} + iy - \cancel{x} + iy$$

$$2iy$$

$$2i \operatorname{Im}(z)$$

$$z_1 + z_2 = z$$

$$|z_1 + z_2|^2 \rightarrow \text{Comp.}$$

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$|z| = \sqrt{x+iy} \quad \begin{matrix} z + \overline{z} = 2 \operatorname{Re}(z) \\ (z_1 + z_2) + (\overline{z_1 + z_2}) = 2 \operatorname{Re}(z_1 + z_2) \end{matrix}$$

$$\underbrace{(z_1 + z_2)}_z \underbrace{(\overline{z_1 + z_2})}_{\overline{z}} = |z_1 + z_2|^2 = |z|^2$$

$$(z_1 + z_2)(\overline{z_1} + \overline{z_2})$$

$$z_1 \overline{z_1} + z_1 \overline{z_2} + z_2 \overline{z_1} + z_2 \overline{z_2}$$

$$\underbrace{|z_1|^2 + |z_2|^2}_{\text{Comp.}} + z_1 \overline{z_2} + \overline{z_1} z_2$$

$$|z_1|^2 + |z_2|^2 + z_1 \overline{z_2} + \overline{z_1} z_2$$

$$|z_1|^2 + |z_2|^2 + 2 \operatorname{Re}(z_1 \overline{z_2})$$

$$\begin{aligned} \overline{z_1 \cdot z_2} \\ \Rightarrow \overline{z_1} \cdot \overline{z_2} \\ \Rightarrow \overline{z_1} \cdot z_2 \end{aligned}$$

$$|z|^2 = z \cdot \overline{z}$$

$$z = x + iy$$

$$\overline{z} = x - iy$$

$$z \cdot \overline{z} = (x + iy)(x - iy)$$

$$= x^2 - (iy)^2$$

$$= x^2 - i^2 y^2 = x^2 + y^2 = |z|^2$$

Find real theta such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is purely real.

If $x + iy = \sqrt{\frac{a + ib}{c + id}}$, **prove that:** $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.

Find the least positive value of n , if $\left(\frac{1+i}{1-i}\right)^n = 1$.

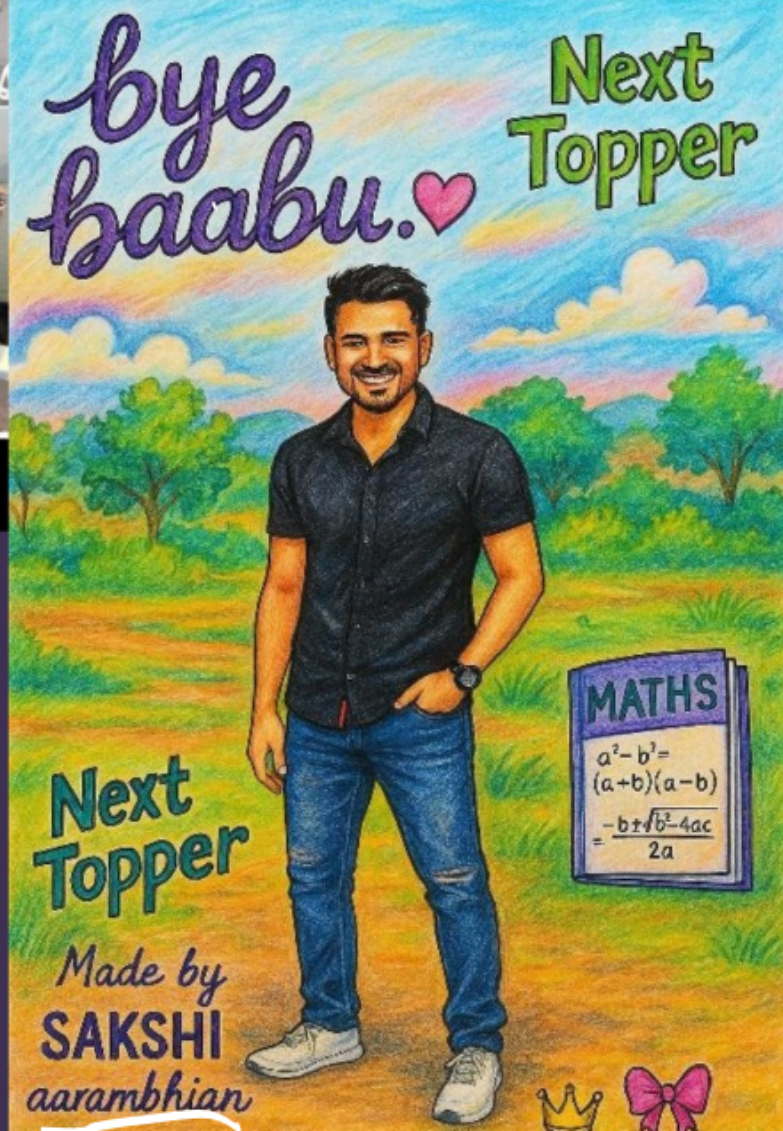
If $a + ib = \frac{c + i}{c - i}$, where c is real, prove that: $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2 - 1}$.

If $(x + iy)^{1/3} = a + ib$, $x, y, a, b \in \mathbb{R}$. Show that

$$\textcircled{i} \quad \frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2) \qquad \textcircled{ii} \quad \frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$$

If $\frac{a + ib}{c + id} = x + iy$, **prove that** $\frac{a - ib}{b - id} = x - iy$ **and** $\frac{a^2 + b^2}{c^2 + d^2} = x^2 + y^2$.

If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$ prove that
 $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$



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Thank you Goodieess !!