LECTURE 03

CLASS XI - PHYSICS

BASIC MATHS AND CALCULUS

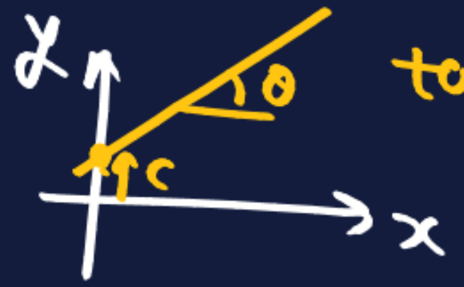
प्रारंभ BATCH

RECAP

lec. 1 — Trigonometry

lec-2 — Algebra

① $y = mx + c$



$\tan\theta = m = \text{Slope}$

② $ax^2 + bx + c = 0$

$$\alpha \text{ OR } \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RECAP

Lec = 2

A.P.

$$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$$

$$S_n = \frac{n}{2}\left[1^{st}\text{ term} + n^{th}\text{ term}\right]$$

• G.P.

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$S_n = \frac{a}{1-r}(1-r^n)$$

If $n \to \infty$

$|r| < 1$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

RECAP

Lec = 2

- Binomial Approximation

$$(1 \pm \square)^n \approx 1 \pm n\square$$

condition

$\square << 1$

Topics to be Covered

- Logarithms & Exponential
- Differentiation

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How’s the JOSH?


दुनिया में कितने प्रकार के
लोग होते है ?

$$\log_{b} x = y$$

(base)

Natural Logarithm (ln)

$$b = e$$

$$y = \log_{e} x = \ln x$$

Common Logarithm (log)

$$b = 10$$

$$y = \log_{10} x$$

Standard Values to remember

Base e	Base 10
$\ln(2) = 0.693$	$\log_{10}(2) = 0.301$
$\ln(3) = 1.09$	$\log_{10}(3) = 0.477$
$\ln 5 = 1.6$	$\log_{10}(5) = 0.699$

Relationship Between Natural and Common Logarithms

$$\ln x = (2.303)\log_{10} x$$

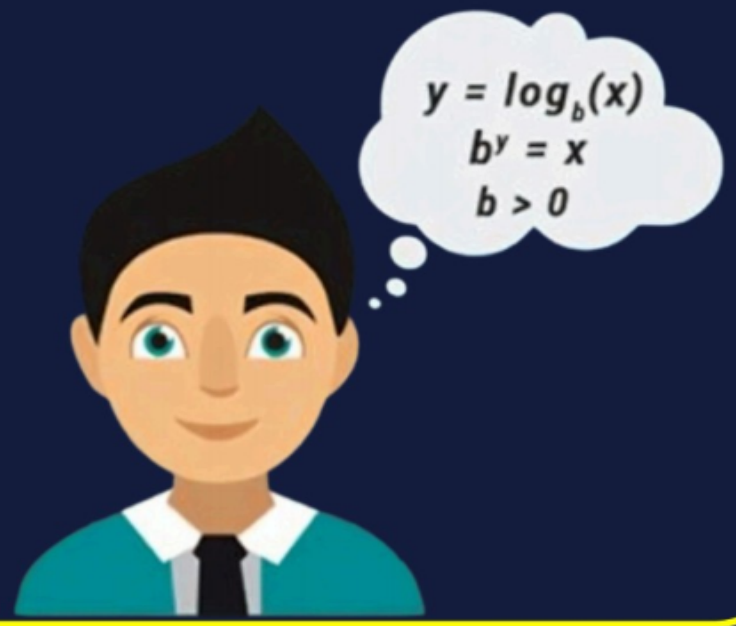
Properties of Logarithms

1. **Product Rule:** $\log(m \cdot n) = \log m + \log n$

2. **Quotient Rule:** $\log\left(\frac{m}{n}\right) = \log m - \log n$

3. **Exponential (Power) Rule:**

$$\log m^n = n \log m$$



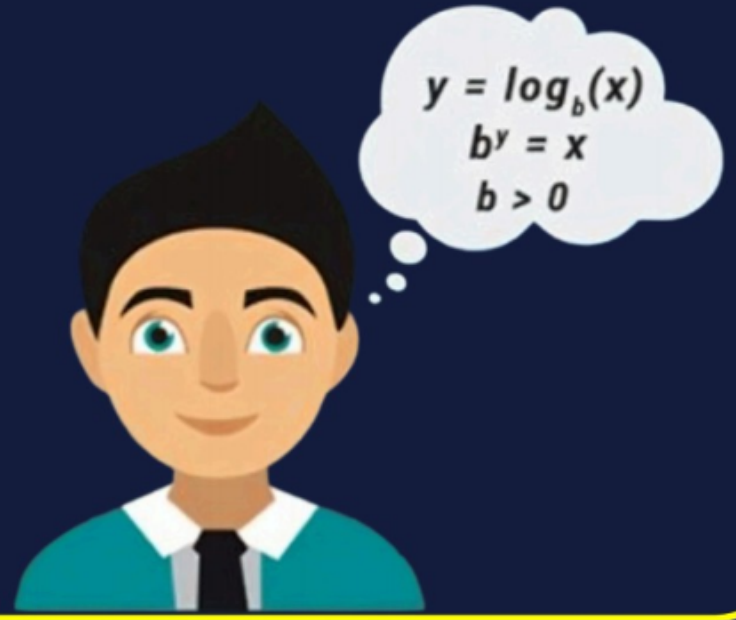
4. Change of Base Rule:

$$\log_b a = \frac{\log_k a}{\log_k b}$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Eg.

$$\log_{10} 5 = \frac{\log_e 5}{\log_e 10} = \frac{\ln 5}{\ln 10}$$



5. Base Switch:

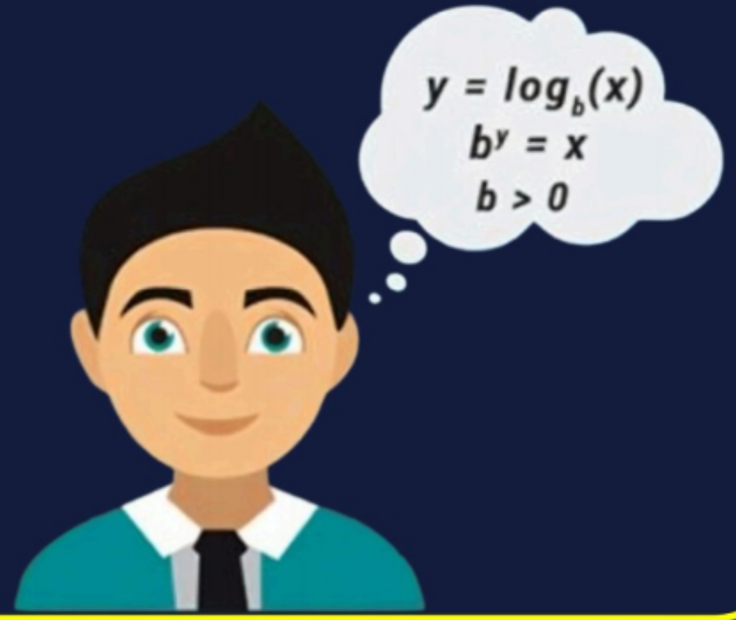
$$\log_b x = \frac{1}{\log_x b}$$

$$\boxed{\log_b a = \frac{1}{\log_a b}}$$

Example:

$$\log_4 16 = \frac{1}{\log_{16} 4}$$

$$\log_9 3 = \frac{1}{\log_3 9}$$



Exponential Function $= (b)^x$

The mathematical constant e = 2.714. $= b$

An exponential function has the form (e)^x or (2.714)^x.

(*e* is the base, and x is the exponent.)

Algbraic	
$y = x^2$	$y = 2^x$
$y = x^3$	$y = 3^x$
$y = x^4$	$y = 4^x$
$y = x^5$	$y = 5^x$

$x = 1, 2, 3, 4, 5, 6 \ldots$

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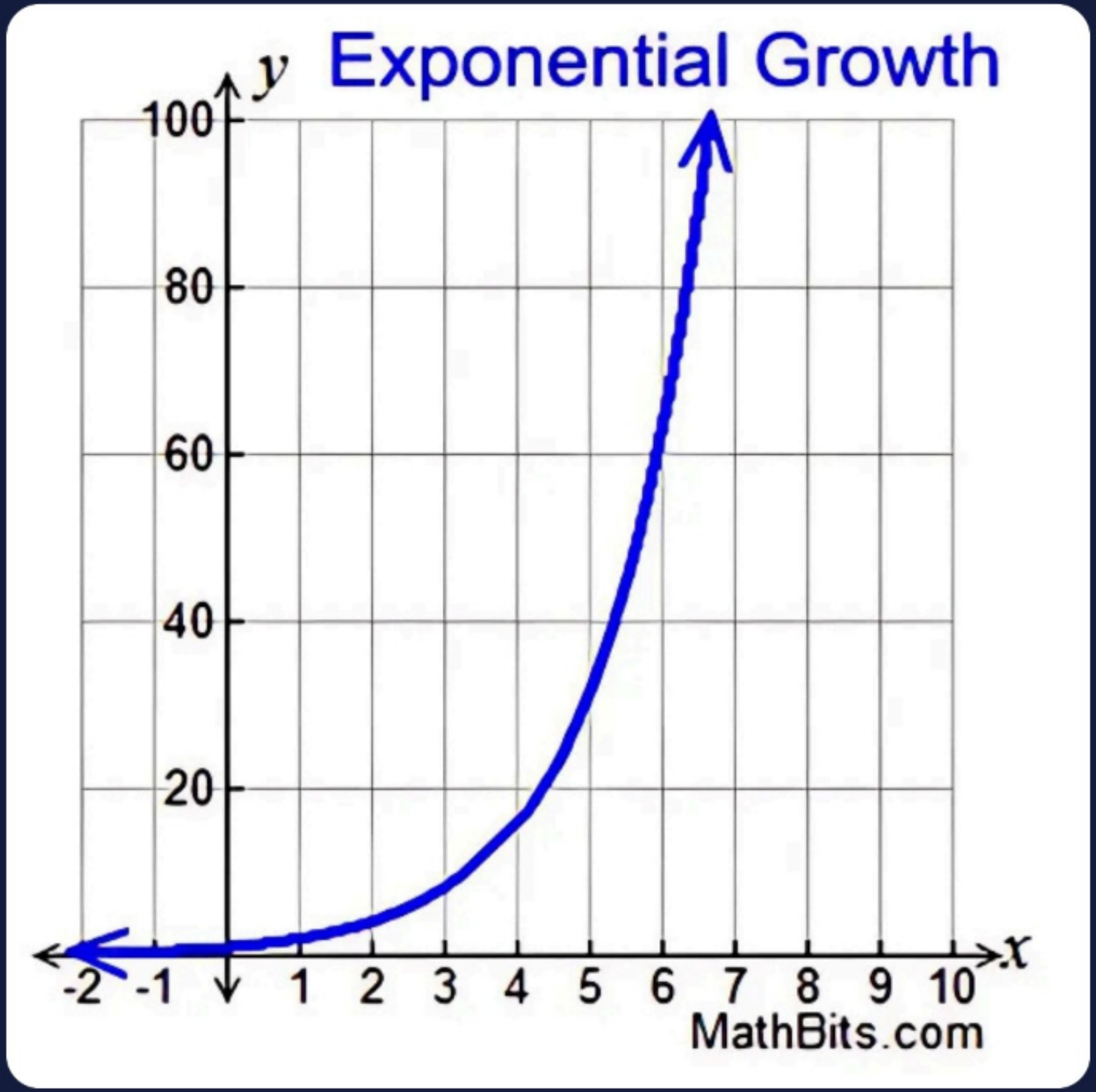


Next Toppers

Calculus

Calculus is a mathematical tool developed to explain and solve problems in physics.

It is divided into two main branches:

1. Differentiation (Derivatives)
2. Integration

Calculus → the mathematical study of continuous change

Differentiation

Types of Functions

$y = f(x)$

1. Constant Function ⟹ Ex: 7, 12, 50, 22
2. Polynomial Function ⟹ Ex: x^4, y^2, x^3+2x
3. Trignometric Function ⟹ Ex: $\cos(x)$, $\sin(y)$
4. Logarithmic Function ⟹ Ex: $y = \log_{10}x$, $\ln x$
5. Exponential Function ⟹ Ex: $y = 2^x$, e^x

Differentiation

Tarika :-

$$y = f(x) \longrightarrow \boxed{\frac{d}{dx}} \longrightarrow \frac{d(y)}{dx} = f'(x)$$

$$y = f(t) \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \frac{dy}{dt} = f'(t)$$

1 Constant Function:

If

$y = f(x) = c$, (where c is any constant)

eg. 2, 5, 15, −25, π

$\boxed{} \rightarrow \frac{dy}{dx} = f'(x) = 0$

② Polynomial Function:

If

$f(x) = x^n,$ where n is any real number

$\frac{dy}{dx} = f'(x)$

$= n x^{n-1}$

Q(1) $y = x^2$

$\frac{dy}{dx} = 2x^{2-1}$

$= 2x$

Q(2) $y = x^{-3/2}$

$\frac{dy}{dx} = \left(-\frac{3}{2}\right) x^{-3/2 - 1}$

$= -\frac{3}{2} x^{-5/2}$

Q(3) $y = t^1 = t$

$\longrightarrow \dfrac{dy}{dt} = (1)\,t^{1-1}$

$= (1)\,t^0$

$= 1$

Q(4) $y = x$

$\longrightarrow \dfrac{dy}{dx} = 1$

Q(5) $y = \dfrac{1}{x}$

$\longrightarrow$

$y = x^{-1}$

$\therefore \dfrac{dy}{dx} = (-1)\,x^{-1-1}$

$= -x^{-2}$

$\dfrac{dy}{dx} = \dfrac{-1}{x^2}$

Q(6) $y = \dfrac{1}{\sqrt{x}}$

$\longrightarrow$

$y = \dfrac{1}{x^{1/2}} = x^{-1/2}$

$\therefore \dfrac{dy}{dx} = \left(-\dfrac{1}{2}\right) x^{-\frac{1}{2}-1}$

$= \left(-\dfrac{1}{2}\right) x^{-3/2}$

$= -\dfrac{1}{2x^{3/2}}$

3 Trigonometric Function:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx}\csc x = -\operatorname{cosec} x \cdot \cot x$$

Properties Of Derivatives

Constant Multiple Formula

$$y = C \cdot x^n \implies \frac{dy}{dx} = C \cdot (n)(x^{n-1})$$

Sum Formula

Difference Formula

$$y = f(x) \pm g(x) \implies \frac{dy}{dx} = f'(x) \pm g'(x)$$

Q(1) $y = 5x^2$

$$\frac{dy}{dx} = 5(2)(x^{2-1})$$
$$= 10x$$

Q(2) $y = \underline{5x^2} + \underline{\underline{2x}}$

$$\frac{d}{dx}(5x^2) = 5(2x) = 10x$$

$$\frac{d}{dx}(2x) = 2(1) = 2$$

$$\Rightarrow \frac{dy}{dx} = 10x + 2$$

Q(3) $y = \sin x - 5\tan x$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx} 5\tan x = 5\sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = \cos x - 5\sec^2 x$$

Properties Of Derivatives

Product Formula

$f(x) = U$

$g(x) = V$

$y = U \cdot V$

$y' = U'V + UV'$

Q(1)

$y = x^2 \sin x$

$U = x^2 \Rightarrow U' = 2x$

$V = \sin x \Rightarrow V' = \cos x$

$y' = \boxed{2x} \sin x + x^2 \boxed{\cos x}$

Q(2)

$$y = (5\sqrt{x})(\tan x)$$

$$y' = \boxed{\frac{5}{2\sqrt{x}}}\tan x + 5\sqrt{x}\,\boxed{\sec^2 x}$$

$$\frac{d}{dx}5\sqrt{x} = \frac{d}{dx}(5)\,x^{1/2}$$

$$= 5\,\frac{d}{dx}x^{1/2}$$

$$= 5\left(\frac{1}{2}\right)x^{1/2-1}$$

$$= \frac{5}{2}x^{-1/2}$$

$$= \frac{5}{2\sqrt{x}}$$

Q(3)

$$y = x^3 \cos x$$

$$y' = \frac{dy}{dx} = \boxed{3x^2} \cos x + x^3 \boxed{(-\sin x)}$$

$$= 3x^2 \cos x - x^3 \sin x$$

Properties Of Derivatives

Quotient Formula

$$y = \frac{U}{V}$$

$$y' = \frac{U'V - UV'}{V^2}$$

Q(1) $y = \frac{\sin\theta}{\cos\theta} = \tan\theta \Rightarrow \frac{dy}{d\theta} = \sec^2\theta$

$$y' = \frac{dy}{d\theta} = \frac{\boxed{\cos\theta}\cos\theta - \sin\theta\boxed{(-\sin\theta)}}{\cos^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

4 Logarithmic Functions

Standard Log Function:

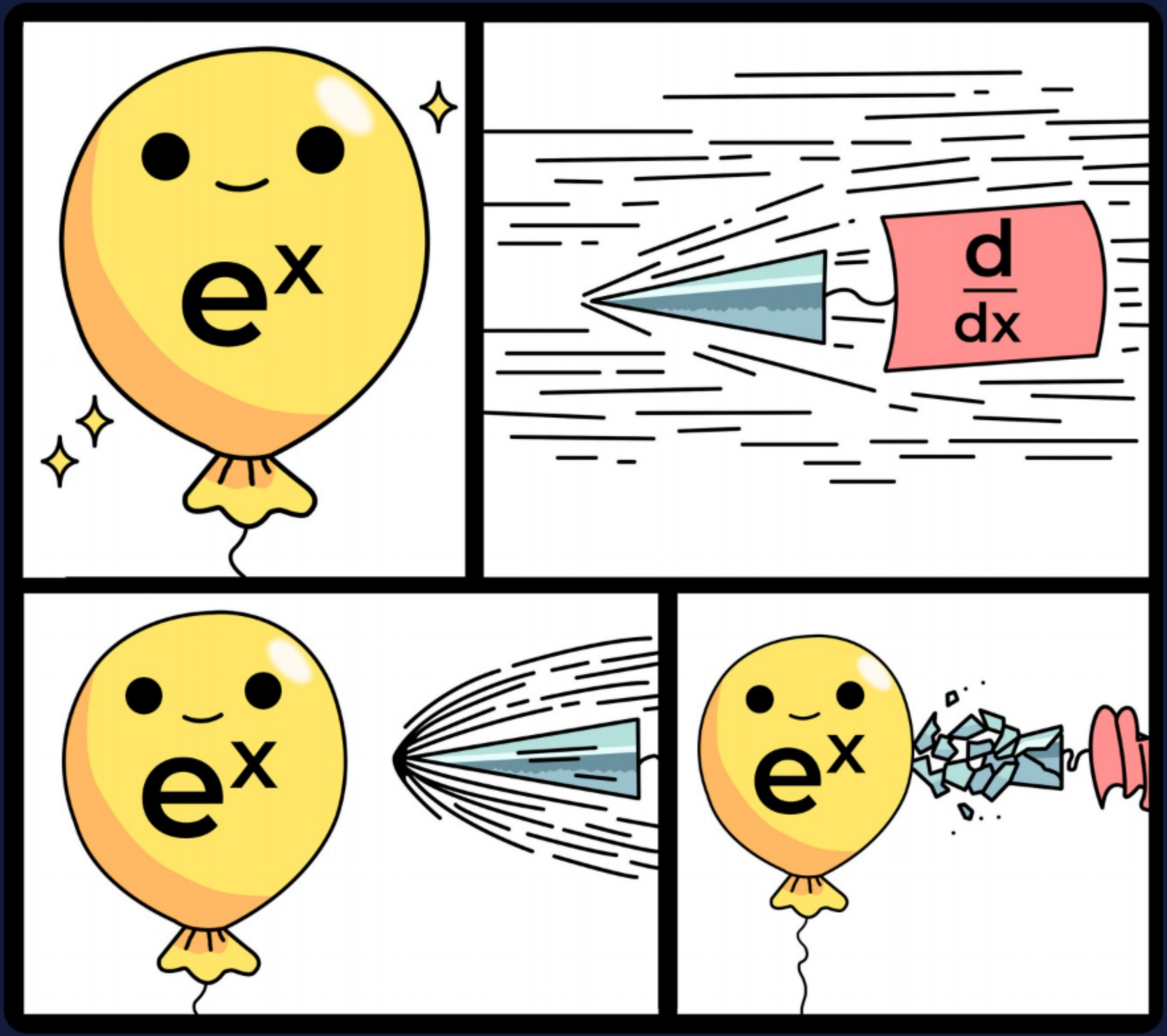
$$y = \log_e x = \ln x$$

$$y = \ln x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$\star \quad y = \log_b(x)$$

$$\frac{dy}{dx} = \frac{1}{x \ln b}$$


e^x
$\frac{d}{dx}$
e^x
e^x

Exponential Functions

$y = e^x$

$\Rightarrow \frac{dy}{dx} = e^x$

$y = b^x$

$\Rightarrow \frac{dy}{dx} = b^x \ln b$

eg. $y = 2^x$

$\frac{dy}{dx} = 2^x \ln(2)$

NEXt LECTURE GOAL

- Applications of Differentiation

(physics
⇒ Feel)

HOME WORK

Example3. Find dy/dx for the following fonctions:

(i) $y = x^5 + x^3 + 10$ (ii) $y = x + \sqrt{x} + \frac{1}{\sqrt{x}}$ (iii) $y = 5x^4 + 3x^{3/2} + 6x$

Example4. Differentiate the following fonctions:

(i) $(3x^2 + 7)(6x + 3)$ (ii) $\frac{x^2 + 1}{x - 2}$ (iii) $\sqrt{4x^2 - 7}$.

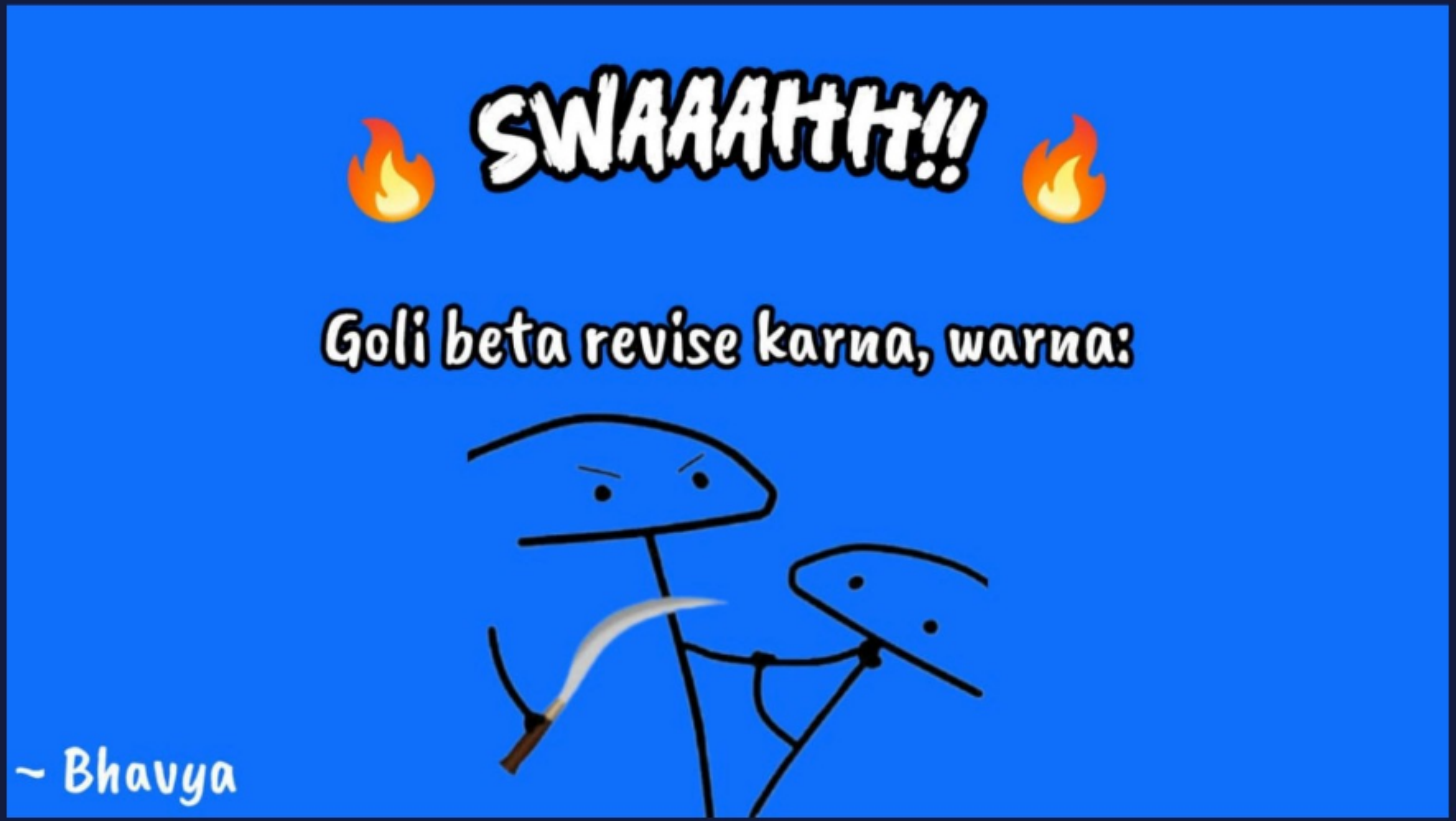
Example5. Find the differential coefficient of the following functions:

(i) $\cos(ax^2 + b)$ (ii) $\tan^3 x$ (iii) $\frac{\sin x}{1 + \cos x}$


SWAAHAAAA !


PYARE GOLI BHAIYA!!
So hello everyone swagat karta hun mere lal bal gopal kaa!!!
Designed by:-GOURAB SAHA


LECTURE ENDS
SWAHAAA!
THANK YOU
Made with love by Akash Gautam


SWAAAHH!!
Goli beta revise karna, warna:
~ Bhavya

SWAHA

STAY CONNECTED

KEEP LEARNING

Thank You