ALWAYS BELIEVE IN YOURSELF

Class 11th

Complex Numbers

Lecture 4

CHAPTER OVERVIEW

INTRODUCTION

CONCEPT OF IOTA

COMPLEX NUMBER

EQUALITY OF COMPLEX NUMBER

OPERATION ON COMPLEX NUMBER

CONJUGATE AND MODULES.

ARGAND PLANE AND POLAR REPRESENTATION

MISCELLANEOUS PROBLEMS

MULTIPLICATIVE INVERSE

$$\frac{1}{z} = z^{-1}$$

$$z = x + iy$$

$$\frac{1}{z} = \frac{1}{x+iy} \rightarrow \text{Deno.}$$

$$\Rightarrow \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$$

$$\frac{1}{z} = \frac{x-iy}{(x+iy)(x-iy)}$$

$$\Rightarrow \frac{x-iy}{x^2-(i^2y^2)}$$

$$\Rightarrow \frac{x-iy}{x^2-(-1y^2)}$$

$$\frac{x-iy}{x^2+y^2}$$

$$x$$

$$\frac{1}{2i-3}$$

$$\Rightarrow \frac{1}{-3+2i} \times \frac{-3-2i}{-3-2i}$$

Prarambh BATCH

Next Toppers

Division of Complex Numbers

$z_1 = x + iy \quad , \quad z_2 = a + ib$

$$\frac{z_1}{z_2} = \frac{x+iy}{a+ib} \times \frac{a-ib}{a-ib}$$

$$\Rightarrow \frac{(x+iy)(a-ib)}{(a+ib)(a-ib)}$$

$$\Rightarrow \frac{(x+iy)(a-ib)}{a^2+b^2}$$

$(a+ib)(a-ib)$

$a^2 - (ib)^2$

$a^2 - (i^2 b^2)$

$a^2 - (-1\, b^2)$

$a^2 + b^2$

$\rightarrow z = a+ib$

$\boxed{\bar{z} = a-ib}$

Conjuguate

$z = x+iy$

$\bar{z} = \overline{x+iy} = x-iy$

$$\frac{1}{z} = \frac{1}{a+ib} \times \frac{a-ib}{a-ib}$$

$$\Rightarrow \frac{a-ib}{a^2+b^2}$$

$z = a+ib \Rightarrow |a+ib|$

$\rightarrow \boxed{|z| = \sqrt{a^2+b^2}}$

Modulus

Modulus $|-2| = 2$

$2+3i$

$$|z| = \sqrt{2^2+3^2}$$

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

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प्रारंभ BATCH

Next Toppers

Express each one of the following in the standard form a + ib .

(i) $\frac{5+4i}{4+5i}$

(ii) $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

(iii) $\frac{1}{1-\cos(\theta)+2i\sin(\theta)}$

(iv) $\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}+i\sqrt{2}\right)}$

$\frac{5+4i}{4+5i} \times \frac{4-5i}{4-5i}$

$\Rightarrow \frac{(5+4i)(4-5i)}{4^2+5^2}$

$\Rightarrow \frac{20-25i+16i-20i^2}{16+25}$

$\Rightarrow \frac{20-9i+20}{41}$

$\Rightarrow \frac{40}{41}-\frac{9i}{41}$

$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

$\Rightarrow \frac{6+9i-4i+6}{2-i+4i+2}$

$\Rightarrow \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$

$\Rightarrow \frac{48-36i+20i+15}{4^2+3^2}$

$\Rightarrow \frac{63-16i}{25}$

प्रारंभ BATCH

Next Toppers

$(1-\cos\theta)^2+(2\sin\theta)^2$

Express each one of the following in the standard form a + ib .

(i) $\dfrac{5+4i}{4+5i}$

(ii) $\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

(iii) $\dfrac{1}{\underbrace{1-\cos(\theta)}_{a}+\underbrace{2i\sin(\theta)}_{b}}$

(iv) $\dfrac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}+i\sqrt{2}\right)}$

$\dfrac{1}{(1-\cos\theta)+2i\sin\theta}\times\dfrac{(1-\cos\theta)-2i\sin\theta}{(1-\cos\theta)-2i\sin\theta}$

$\Rightarrow \dfrac{1-\cos\theta-2i\sin\theta}{(1-\cos\theta)^2+(2\sin\theta)^2}$

$\Rightarrow \dfrac{1-\cos\theta-2i\sin\theta}{1^2+\cos^2\theta-2\cos\theta+4\sin^2\theta}$

$\Rightarrow \dfrac{1-\cos\theta-2i\sin\theta}{1-2\cos\theta+\cos^2\theta+4\sin^2\theta}$

$\dfrac{(1-\cos\theta)-2i\sin\theta}{1-2\cos\theta+\cos^2\theta+\sin^2\theta+3\sin^2\theta}$

$\Rightarrow \dfrac{(1-\cos\theta)-2i\sin\theta}{1-2\cos\theta+1+3\sin^2\theta}$

$\Rightarrow \dfrac{(1-\cos\theta)-2i\sin\theta}{2-2\cos\theta+3\sin^2\theta}$

प्रारंभ BATCH

Next Toppers

Express each one of the following in the standard form a + ib .

(i) $\dfrac{5+4i}{4+5i}$

(ii) $\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

(iii) $\dfrac{1}{1-\cos(\theta)+2i\sin(\theta)}$

(iv) $\dfrac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$

$$\frac{3^2-(i\sqrt{5})^2}{\cancel{\sqrt{3}}+\sqrt{2}i-\cancel{\sqrt{3}}+\sqrt{2}i}$$

$$\Rightarrow \frac{9-i^2\times 5}{2\sqrt{2}i}$$

$$\Rightarrow \frac{9+5}{2\sqrt{2}i}$$

$$\Rightarrow \frac{\cancel{14}\,7}{\cancel{2}\sqrt{2}i}$$

$$\frac{7}{\sqrt{2}i}\times\frac{\sqrt{2}i}{\sqrt{2}i}$$

$$\frac{7\sqrt{2}i}{2i^2}$$

$$\Rightarrow \frac{7\sqrt{2}i}{2(-1)}$$

$$\Rightarrow \frac{7\sqrt{2}i}{-2}$$

$$\Rightarrow -\frac{7\sqrt{2}i}{2}$$

If z_1, z_2 are $1 - i, -2 + 4i$ respectively, find $\mathrm{Im}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$.

$z_1 = 1 - i \longrightarrow \bar{z}_1 = 1 + i$

$z_2 = -2 + 4i$

$\frac{z_1 z_2}{\bar{z}_1} = \frac{(1-i)(-2+4i)}{1+i}$

$= \frac{-2+4i+2i-4i^2}{1+i}$

$\frac{-2+6i-4(-1)}{1+i}$

$\frac{-2+6i+4}{1+i}$

$\Rightarrow \frac{2+6i}{1+i} \times \frac{(1-i)}{1-i}$

$\frac{2-2i+6i+6}{1^2+1^2}$

$\frac{8+4i}{2}$

$\frac{8}{2} + \frac{4}{2} i$

$4 + 2i$

$\mathrm{Im}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = 2$

प्रारंभ BATCH

Next Toppers

Find the conjugate of $\dfrac{1}{3+4i}$. $\rightarrow 3-4i \times \quad 3+4i$

$z = \dfrac{1}{3+4i} \times \dfrac{3-4i}{3-4i}$

$\Rightarrow \dfrac{3-4i}{3^2+4^2}$

$\Rightarrow \dfrac{3-4i}{9+16}$

$z \Rightarrow \dfrac{3-4i}{25}$

$z = \dfrac{3-4i}{25}$

$\bar{z} = \overline{\dfrac{3-4i}{25}} = \dfrac{3+4i}{25}$

Find real values of x and y for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

$\boxed{y = -4}$ $\boxed{x = \pm 1}$

$\rightarrow z_1 = -3 + ix^2y$

$\rightarrow z_2 = x^2 + y + 4i$

$\rightarrow \bar{z}_1 = z_2$ | $\bar{z}_2 = z_1$

$\bar{z}_1 = \overline{-3 + ix^2y}$

$= -3 - ix^2y$

$\boxed{\bar{z}_1 = z_2}$

$-3 - ix^2y = x^2 + y + 4i$

$-3 = x^2 + y$ | $-x^2y = 4$

$x^2 = -\frac{4}{y}$

$-3 = \frac{-4}{y} + y$

$-3 = \frac{-4 + y^2}{y}$

$-3y = -4 + y^2$

$-3y = -4 + y^2$

$0 = y^2 + 3y - 4$

$y^2 + 4y - y - 4 = 0$

$y(y+4) - 1(y+4) = 0$

$(y+4)(y-1) = 0$

$y = -4$ $y = 1$

$y = -4$

$x^2 = \frac{+4}{+4}$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

$y = 1$

$x^2 = -\frac{4}{1}$

$x^2 = -4$

$\boxed{x = \pm\sqrt{-4} = \pm 2i}$

Find the multiplicative inverse of the following complex numbers:

(i) $3 + 2i$

(ii) $(2 + \sqrt{3}i)^2$

$z = 3+2i$

$z^{-1} = \dfrac{1}{3+2i} \times \dfrac{3-2i}{3-2i}$

$\Rightarrow \dfrac{3-2i}{3^2-(2i)^2}$

$\dfrac{3-2i}{9-4i^2}$

$\dfrac{3-2i}{9-4(-1)}$

$\Rightarrow \dfrac{3-2i}{9+4}$

$\Rightarrow \dfrac{3-2i}{13}$

$(2+\sqrt{3}i)^2 \Rightarrow 2^2+(\sqrt{3}i)^2+2\times 2\times\sqrt{3}i$

$\Rightarrow 4-3+4\sqrt{3}i$

$\Rightarrow 1+4\sqrt{3}i$

$z^{-1} = \dfrac{1}{1+4\sqrt{3}i} \times \dfrac{1-4\sqrt{3}i}{1-4\sqrt{3}i}$

$\Rightarrow \dfrac{1-4\sqrt{3}i}{1^2-(4\sqrt{3}i)^2}$

$\Rightarrow \dfrac{1-4\sqrt{3}i}{1-16\times 3i^2}$

$\dfrac{1-4\sqrt{3}i}{1-48(-1)}$

$\dfrac{1-4\sqrt{3}i}{1+48}$

$\Rightarrow \dfrac{1-4\sqrt{3}i}{49}$